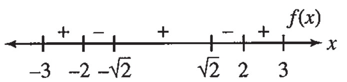
8. $\dfrac{x^2-2}{x^2-4}=0$ when $x=\pm\sqrt{2}$ and is undefined when $x=\pm 2$.

Test one point on each subinterval: for

$x=-\dfrac{5}{2}, \dfrac{x^2-2}{x^2-4}=\dfrac{17}{9}$; for $x=-1.9, \dfrac{x^2-2}{x^2-4}\approx-4.13$;

for $x=0, \dfrac{x^2-2}{x^2-4}=\dfrac{1}{2}$; for $x=1.9, \dfrac{x^2-2}{x^2-4}\approx-4.13$;

and for $x=\dfrac{5}{2}, \dfrac{x^2-2}{x^2-4}=\dfrac{17}{9}$. The function changes sign at -2, $-\sqrt{2}$, $\sqrt{2}$ and 2. The graph is

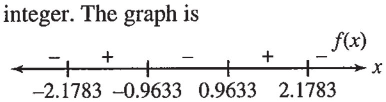


9. $\sec\left(1+\sqrt{1-\sin^2 x}\right)=\dfrac{1}{\cos(1+|\cos x|)}$ is undefined when

$x\approx 0.9633+k\pi$ or $2.1783+k\pi$ for any integer k.

Test for $x=0$: $\sec\left(1+\sqrt{1-\sin^2 0}\right)\approx-2.4030$.

Test for $x=\pm 1$: $\sec\left(1+\sqrt{1-\sin^2 1}\right)\approx 32.7984$. The sign alternates over successive subintervals. The function changes sign at $0.9633+k\pi$ or $2.1783+k\pi$, where k is an integer. The graph is

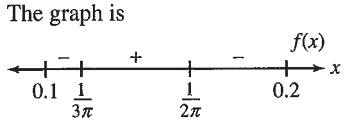


10. On the interval, $\sin\left(\dfrac{1}{x}\right)=0$ when $x=\dfrac{1}{3\pi}$ or $\dfrac{1}{2\pi}$. Test one point on each subinterval: for $x=0.1$, $\sin\left(\dfrac{1}{x}\right)\approx-0.54$;

for $x=0.15$, $\sin\left(\dfrac{1}{x}\right)\approx 0.37$; and for $x=0.2$,

$\sin\left(\dfrac{1}{x}\right)\approx-0.96$. The graph changes sign at $\dfrac{1}{3\pi}$, and $\dfrac{1}{2\pi}$.

The graph is



Section 7.1 Exercises

1. **(a)** Right when $v(t)>0$, which is when $\cos t>0$, i.e., when $0\le t<\dfrac{\pi}{2}$ or $\dfrac{3\pi}{2}<t\le 2\pi$. Left when $\cos t<0$, i.e., when $\dfrac{\pi}{2}<t<\dfrac{3\pi}{2}$. Stopped when $\cos t=0$, i.e., when $t=\dfrac{\pi}{2}$ or $\dfrac{3\pi}{2}$.

(b) Displacement =

$\int_0^{2\pi}5\cos t\,dt=5[\sin t]_0^{2\pi}=5[\sin 2\pi-\sin 0]=0$

Final position $=3+0=3$

(c) Distance $=\int_0^{2\pi}|5\cos t|\,dt$

$=\int_0^{\pi/2}5\cos t\,dt+\int_{\pi/2}^{3\pi/2}-5\cos t\,dt+\int_{3\pi/2}^{2\pi}5\cos t\,dt$

$=5+10+5=20$

2. **(a)** Right when $v(t)>0$, which is when $\sin 3t>0$, i.e., when $0<t<\dfrac{\pi}{3}$. Left when $\sin 3t<0$, i.e., when $\dfrac{\pi}{3}<t\le\dfrac{\pi}{2}$. Stopped when $\sin 3t=0$, i.e., when $t=0$ or $\dfrac{\pi}{3}$.

(b) Displacement $=\int_0^{\pi/2}6\sin 3t\,dt=6\left[-\dfrac{1}{3}\cos 3t\right]_0^{\pi/2}$

$=-2\left[\cos\dfrac{3\pi}{2}-\cos 0\right]=2$

Final position $=3+2=5$

(c) Distance $=\int_0^{\pi/2}|6\sin 3t|\,dt$

$=\int_0^{\pi/3}6\sin 3t\,dt+\int_{\pi/3}^{\pi/2}-6\sin 3t\,dt=4+2=6$

3. **(a)** Right when $v(t)=49-9.8t>0$, i.e., when $0\le t<5$. Left when $49-9.8t<0$, i.e., when $5<t\le 10$. Stopped when $49-9.8t=0$, i.e., when $t=5$.

(b) Displacement $=\int_0^{10}(49-9.8t)\,dt$

$=\left[49t-4.9t^2\right]_0^{10}=49[(10-10)-0]=0$

Final position $=3+0=3$.

(c) Distance $=\int_0^{10}|49-9.8t|\,dt$

$=\int_0^5(49-9.8t)\,dt+\int_5^{10}(-49+9.8t)\,dt$

$=122.5+122.5=245$

4. **(a)** Right when $v(t)=6t^2-18t+12=6(t-1)(t-2)>0$, i.e., when $0\le t<1$. Left when $6(t-1)(t-2)<0$, i.e., when $1<t<2$. Stopped when $6(t-1)(t-2)=0$, i.e., when $t=1$, or 2.

(b) Displacement $=\int_0^2(6t^2-18t+12)\,dt$

$=\left[2t^3-9t^2+12t\right]_0^2=[(16-36+24)-0]=4$

Final position $=3+4=7$

(c) Distance $=\int_0^2|6t^2-18t+12|\,dt$

$=\int_0^1(6t^2-18t+12)\,dt+\int_1^2(-6t^2+18t-12)\,dt$

$=5+1=6$

5. **(a)** Right when $v(t) > 0$, which is when $\sin t \neq 0$ and $\cos t > 0$, i.e., when $0 < t < \frac{\pi}{2}$ or $\frac{3\pi}{2} < t < 2\pi$. Left when $\sin t \neq 0$ and $\cos t < 0$, i.e., when $\frac{\pi}{2} < t < \pi$ or $\pi < t < \frac{3\pi}{2}$. Stopped when $\sin t = 0$ or $\cos t = 0$, i.e., when $t = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$, or 2π.

(b) Displacement $= \int_0^{2\pi} 5\sin^2 t \cos t\, dt = 5\left[\frac{1}{3}\sin^3 t\right]_0^{2\pi}$

$= 5[0 - 0] = 0$

Final position $= 3 + 0 = 3$

(c) Distance $= \int_0^{2\pi} \left|5\sin^2 t \cos t\right| dt$

$= \int_0^{\pi/2} 5\sin^2 t \cos t\, dt + \int_{\pi/2}^{3\pi/2} -5\sin^2 t \cos t\, dt + \int_{3\pi/2}^{2\pi} 5\sin^2 t \cos t\, dt$

$= \frac{5}{3} + \frac{10}{3} + \frac{5}{3} = \frac{20}{3}$

6. **(a)** Right when $v(t) > 0$, which is when $4 - t > 0$, i.e., when $0 \le t < 4$. Left: never, since $\sqrt{4-t}$ cannot be negative. Stopped when $4 - t = 0$, i.e., when $t = 4$.

(b) Displacement $= \int_0^4 \sqrt{4-t}\, dt = \left[-\frac{2}{3}(4-t)^{3/2}\right]_0^4$

$= -\frac{2}{3}\left[0 - 8\right] = \frac{16}{3}$

Final position $= 3 + \frac{16}{3} = \frac{25}{3}$

(c) Distance $= \int_0^4 \sqrt{4-t}\, dt = \frac{16}{3}$

7. **(a)** Right when $v(t) > 0$, which is when $\cos t > 0$, i.e., when $0 \le t < \frac{\pi}{2}$ or $\frac{3\pi}{2} < t \le 2\pi$. Left when $\cos t < 0$, i.e., when $\frac{\pi}{2} < t < \frac{3\pi}{2}$. Stopped when $\cos t = 0$, i.e., when $t = \frac{\pi}{2}$ or $\frac{3\pi}{2}$.

(b) Displacement $= \int_0^{2\pi} e^{\sin t} \cos t\, dt = \left[e^{\sin t}\right]_0^{2\pi}$

$= [e^0 - e^0] = 0$

Final position $= 3 + 0 = 3$

(c) Distance $= \int_0^{2\pi} \left|e^{\sin t} \cos t\right| dt = \int_0^{\pi/2} e^{\sin t} \cos t\, dt +$

$\int_{\pi/2}^{3\pi/2} -e^{\sin t} \cos t\, dt + \int_{3\pi/2}^{2\pi} e^{\sin t} \cos t\, dt$

$= (e-1) + \left(e - \frac{1}{e}\right) + \left(1 - \frac{1}{e}\right) = 2e - \frac{2}{e} \approx 4.7$

8. **(a)** Right when $v(t) > 0$, which is when $0 < t \le 3$. Left: never, since $v(t)$ is never negative. Stopped when $t = 0$.

(b) Displacement $= \int_0^3 \frac{t}{1+t^2}\, dt = \left[\frac{1}{2}\ln(1+t^2)\right]_0^3$

$= \frac{1}{2}\left[\ln(10) - \ln(1)\right] = \frac{\ln 10}{2} \approx 1.15$

Final position $= 3 + \frac{\ln 10}{2} \approx 4.15$

(c) Distance $= \int_0^3 \frac{t}{1+t^2}\, dt = \frac{\ln 10}{2} \approx 1.15$

9. **(a)** $v(t) = \int a(t)\, dt = t + 2t^{3/2} + C$, and since $v(0) = 0$,

$v(t) = t = +2t^{3/2}$. Then $v(9) = 9 + 2(27) = 63$ mph.

(b) First convert units:

$t + 2t^{3/2}$ mph $= \frac{t}{3600} + \frac{t^{3/2}}{1800}$ mi / sec. Then

Distance $= \int_0^9 \left(\frac{t}{3600} + \frac{t^{3/2}}{1800}\right) dt$

$= \left[\frac{t^2}{7200} + \frac{t^{5/2}}{4500}\right]_0^9 = \left[\left(\frac{9}{800} + \frac{27}{500}\right) - 0\right] = 0.06525$ mi

$= 344.52$ ft.

10. **(a)** Displacement $= \int_0^4 (t-2)\sin t\, dt$

$= \left[\sin t - t\cos t + 2\cos t\right]_0^4$

$= \left[(\sin 4 - 4\cos 4 + 2\cos 4) - 2\right] \approx -1.44952$ m

(b) Because the velocity is negative for $0 < t < 2$, positive for $2 < t < \pi$, and negative for $\pi < t \le 4$,

Distance $= \int_0^2 -(t-2)\sin t\, dt + \int_2^{\pi} (t-2)\sin t\, dt + \int_{\pi}^4 -(t-2)\sin t\, dt$

$= [(2 - \sin 2) + (\pi - \sin 2 - 2) + (\pi + 2\cos 4 - \sin 4 - 2]$

$= 2\pi + 2\cos 4 - 2\sin 2 - \sin 4 - 2 \approx 1.914$ m.

11. **(a)** $v(t) = \int a(t)\, dt = \int -32\, dt = -32t + C_1$, where $C_1 = v(0) = 90$.

Then $v(3) = -32(3) + 90 = -6$ ft / sec.

(b) $s(t) = \int v(t)\, dt = -16t^2 + 90t + C_2$, where $C_2 = s(0) = 0$. Solve $s(t) = 0$:

$-16t^2 + 90t = 2t(-8t + 45) = 0$

when $t = 0$ or $t = \frac{45}{8} = 5.625$ sec.

The projectile hits the ground at 5.625 sec.

(c) Since starting height = ending height, Displacement = 0.

11. Continued

(d) Max. Height $= s\left(\frac{5.625}{2}\right)$

$$= -16\left(\frac{5.625}{2}\right)^2 + 90\left(\frac{5.625}{2}\right) = 126.5625, \text{ and}$$

Distance = 2(Max. Height) = 253.125 ft.

12. Displacement $= \int_0^c v(t)\,dt = -4+5-24 = -23\text{cm}$

13. Total distance $= \int_0^c |v(t)|\,dt = 4+5+24 = 33\text{cm}$

14. At $t = a$, $s = s(0) + \int_0^a v(t)\,dt = 15-4 = 11.$

At $t = b$, $s = s(0) + \int_0^b v(t)\,dt = 15-4+5 = 16.$

At $t = c$, $s = s(0) + \int_0^c v(t)\,dt = 15-4+5-24 = -8.$

15. At $t = a$, where $\frac{dv}{dt}$ is at a maximum (the graph is steepest upward).

16. At $t = c$, where $\frac{dv}{dt}$ is at a maximum (the graph is steepest upward).

17. Distance = Area under curve $= 4\left(\frac{1}{2} \bullet 1 \bullet 2\right) = 4$

(a) Final position = Initial position + Distance = 2 + 4 = 6; ends at $x = 6$.

(b) 4 meters

18. (a) Positive and negative velocities cancel; the sum of signed areas is zero. Starts and ends at $x = 2$.

(b) Distance = Sum of positive areas = 4(1 • 1) = 4 meters

19. (a) Final position $= 2 + \int_0^7 v(t)\,dt$

$$= 2 - \frac{1}{2}(1)(2) + \frac{1}{2}(1)(2) + 1(2) + \frac{1}{2}(2)(2) - \frac{1}{2}(2)(1) = 5;$$

end at $x = 5$.

(b) $\int_0^7 |v(t)|\,dt = \frac{1}{2}(1)(2) + \frac{1}{2}(1)(2) + 1(2) + \frac{1}{2}(2)(2) + \frac{1}{2}(2)(1)$

$= 7$ meters

20. (a) Final position $= 2 + \int_0^{10} v(t)$

$$= 2 + \frac{1}{2}(2)(3) - \frac{1}{2}(1)(3) - (3)(3) - \frac{1}{2}(1)(3) + \frac{1}{2}(3)(3) = -2.5;$$

ends at $x = -2.5$

(b) Distance $\int_0^{10} |v(t)|\,dt$

$$= \frac{1}{2}(2 \cdot 3) + \frac{1}{2}(1)(3) + 3(3) + \frac{1}{2}(1)(3) + \frac{1}{2}(3)(3)$$

= 19.5 meters

21. $\int_0^{10} 27.08 \bullet e^{t/25}\,dt = 27.08\left[25e^{t/25}\right]_0^{10}$

$= 27.08\left[25e^{0.4} - 25\right]$

≈ 332.965 billion barrels

22. $\int_0^{24}\left[3.9 - 2.4\sin\left(\frac{\pi t}{12}\right)\right]dt = \left[3.9t + \frac{28.8}{\pi}\cos\left(\frac{\pi t}{12}\right)\right]_0^{24}$

$= \left[\left(93.6 + \frac{28.8}{\pi}\right) - \frac{28.8}{\pi}\right]dt = 93.6$ kilowatt-hours

23. (a) solve $10{,}000\,(2-r) = 0$: $r = 2$ miles.

(b) Width $= \Delta r$; Length $= 2\pi r$; Area $= 2\pi r\Delta r$

(c) Population = Population density × Area

(d) $\int_0^2 10{,}000(2-r)(2\pi r)\,dr = 20{,}000\pi\int_0^2 \left(2r - r^2\right)dr$

$$= 20{,}000\pi\left[r^2 - \frac{1}{3}r^3\right]_0^2 = 20{,}000\pi\left[\left(4 - \frac{8}{3}\right) - 0\right]$$

$$= \frac{80{,}000}{3}\pi \approx 83{,}776$$

24. (a) Width $= \Delta r$, Length $= 2\pi r$: Area $= 2\pi r\Delta r$

(b) Volume per second

= Inches per second × Cross section area

$$8\left(10 - r^2\right)\frac{\text{in.}}{\text{sec}} \bullet (2\pi r)\Delta r\,\text{in}^2 = \text{flow in}\,\frac{\text{in}^3}{\text{sec}}$$

(c) $\int_0^3 8(10-r^2)(2\pi r)\,dr = 16\pi\int_0^3 (10r - r^3)\,dr$

$$= 16\pi\left[5r^2 - \frac{1}{4}r^4\right]_0^3 = 16\pi\left[\left(45 - \frac{81}{4}\right) - 0\right]$$

$$= 396\pi\,\frac{\text{in}^3}{\text{sec}} \approx 1244.07\,\frac{\text{in}^3}{\text{sec}}$$

25. (a) Sum of numbers in Sales column = 797.5 thousand

(b) Enter the table in a graphing calculator and use QuadReg: $B(x) = 1.6x^2 + 2.3x + 5.0$.

(c) $\int_0^{11} \left(1.6x^2 + 2.3x + 5.0\right)dx$

$$= \left[\frac{1.6}{3}x^3 + \frac{2.3}{2}x^2 + 5.0x\right]_0^{11}$$

≈ 904.02 thousand

(d) The answer in (a) corresponds to the area of left hand rectangles. These rectangles lie under the curve $B(x)$. The answer in (c) corresponds to the area under the curve. This area is greater than the area of the rectangles.

26. (a) $\int_{-0.5}^{10.5}\left(1.6x^2+2.3x+5.0\right)dr$

$$=\left[\frac{1.6}{3}x^3+\frac{2.3}{2}x^2+5.0x\right]_{-0.5}^{10.5}\approx 798.97 \text{ thousand}$$

(b) The answer in (a) corresponds to the area of rectangles whose heights are the actual sales ("midpoint rectangles"). The curve now gives a better approximation since part of each rectangle is above the curve and part is below.

27. Treat 6 P.M. as 18 o'clock:

$$\frac{b-a}{2n}\left[f(x_0)+\sum_{i=1}^{n-1}2f(x_i)+f(x_n)\right]$$

$$=\frac{18-8}{2(10)}[120+2(110)+2(115)+2(119)+2(120)$$
$$+2(120)+2(115)+2(112)+2(110)+(121)]$$
$$=1156.5$$

28. (Answer may vary.)
Plot the speeds vs. time. Connect the points and find the area under the line graph. The definite integral also gives the area under the curve.

29. $F(x) = kx$; $6 = k(3)$, so $k = 2$ and $F(x) = 2x$.

(a) $F(9) = 2(9) = 18\text{N}$

(b) $W=\int_0^9 F(x)dx=\int_0^9 2x\,dx=\left[x^2\right]_0^9=81\text{N}\cdot\text{cm}$

30. $F(x) = kx$; $10{,}000 = k(1)$, so $k = 10{,}000$.

(a) $W=\int_0^d kx\,dx=\left[\frac{1}{2}kx^2\right]_0^d=\frac{1}{2}kd^2=\frac{1}{2}(10{,}000)(0.5)^2$

$=1250$ inch-pounds

(b) For total distance: $W=\frac{1}{2}(10{,}000)(1)^2=5000$

For second half of distance:

$W = 5000 - 1250 = 3750$ inch-pounds

31. False. The displacement is the integral of the velocity from $t = 0$ to $t = 5$ and is positive, since the region that is under the graph and above the horizontal axis is larger than the region that is above the graph and below the horizontal axis.

32. True. Since the velocity is positive, the integral of the velocity is equal to the integral of its absolute value, which is the total distance traveled.

33. C. To the nearest whole square, the area under the curve covers 12 grid squares. $(12)(50)(6) = 3600$.

34. D. $5+\frac{15}{10}\left(4+2(8)+2(6)+2(9)+2(10)+10\right)=125.$

35. B. $\int_0^{60}\left(12+6\cos\left(\frac{t}{\pi}\right)\right)dt=12t+6\pi\sin\left(\frac{t}{\pi}\right)\Big|_0^{60}\approx 725.$

36. A. $\int_0^{10}20e^{-0.5t}dt=-40e^{-0.5t}\Big|_0^{10}=40$

37. $\frac{(12-0)}{2(12)}[0.04+2(0.04)+2(0.05)+2(0.06)+2(0.05)$
$+2(0.04)+2(0.04)+2(0.05)+2(0.04)$
$+2(0.06)+2(0.06)+2(0.05)+0.05]=0.585$

The overall rate, then, is $\frac{0.585}{12}=0.04875.$

38. $\frac{(12-0)}{2(12)}[3.6+2(4.0)+2(3.1)+2(2.8)+2(2.8)+2(3.2)$
$+2(3.3)+2(3.1)+2(3.2)+2(3.4)+2(3.4)$
$+2(3.9)+4.0]=40$ thousandths or 0.040

39. (a) $\bar{x}=\frac{M_y}{M}=\frac{\Sigma m_k x_k}{\Sigma m_k}$. Taking $dm=\delta dA$ as m_k and letting $dA\to 0, k\to\infty$ yields $\frac{\int x\,dm}{\int dm}$.

(b) $\bar{y}=\frac{M_k}{M}=\frac{\sum m_k y}{\sum m_k}$. Taking $dm=\delta dA$ as m_k and letting $dA\to 0, k\to\infty$ yields $\frac{\int y\,dm}{\int dm}$.

40. By symmetry, $\bar{x}=0$. For $\bar{y}$, use horizontal strips:

$$\bar{y}=\frac{\int y\,dm}{\int dm}=\frac{\int y\delta dA}{\int \delta dA}=\frac{\int y\,dA}{\int dA}$$
$$=\frac{\int_0^4 y(2\sqrt{y})\,dy}{\int_0^4 2\sqrt{y}\,dy}$$
$$=\frac{2\left[\frac{2}{5}y^{5/2}\right]_0^4}{2\left[\frac{2}{3}y^{3/2}\right]_0^4}$$
$$=\frac{12}{5}$$

41. By symmetry, $\bar{y}=0$. For $\bar{x}$, use vertical strips:

$$\bar{x}=\frac{\int x\,dm}{\int dm}=\frac{\int x\delta dA}{\int \delta dA}=\frac{\int x\,dA}{\int dA}$$
$$=\frac{\int_0^2 x(2x)\,dx}{\int_0^2 2x\,dx}$$
$$=\frac{\left[\frac{2}{3}x^3\right]_0^2}{\left[x^2\right]_0^2}$$
$$=\frac{4}{3}$$