Section 10.2 Exercises

1. $(2, 3)-(0, 0)=\langle 2,3\rangle$

2. $(0, 0)-(2, 3)=\langle -2,-3\rangle$

3. $(2, -1)-(1, 3)=\langle 1, -4\rangle$

4. $P=\frac{-4+2}{2}, \frac{3-1}{2}=(-1, 1)$

$(-1, 1)-(0, 0)=\langle -1, 1\rangle$

5. $|v|=\sqrt{2^2+2^2}=\sqrt{8}$

$\tan\theta=\frac{\sqrt{2}}{2}, \theta=45°$

6. $|v|=\sqrt{\left(-\sqrt{2}\right)^2+\sqrt{2}^2}=2$

$\tan\theta=-\frac{\sqrt{2}}{\sqrt{2}},\ \theta=135°$

7. $|v|=\sqrt{\sqrt{3}^2+1^2}=2$

$\tan\theta=\frac{1}{\sqrt{3}}, \theta=30°$

8. $|v|=\sqrt{-2^2+\left(-2\sqrt{3}\right)^2}=4$

$\tan\theta=\frac{-2\sqrt{3}}{2}, \theta=240°$

9. $|v|=\sqrt{(-5)^2+0^2}=5$

$\tan\theta=\frac{0}{-5},\ \theta=180°$

10. $|v|=\sqrt{0^2+4^2}=4$

$\cos\theta=\frac{0}{4},\ \theta=90°$

11. $x=4\cos(180)=-4$

$y=4\sin(180)=0$

$\langle -4, 0\rangle$

12. $x=6\cos(270)=0$

$y=6\sin(270)=-6$

$\langle 0,-6\rangle$

13. $x=5\cos(100)=-0.868$

$y=5\sin(100)=4.924$

$\langle -0.868, 4.924\rangle$

14. $x=13\cos(200)=-12.216$

$y=13\sin(200)=-4.446$

$\langle -12.216,-4.446\rangle$

15. $x=3\sqrt{2}\cos\left(\frac{180}{\pi}\frac{\pi}{4}\right)=3$

$y=3\sqrt{2}\sin\left(\frac{180}{\pi}\frac{\pi}{4}\right)=3$

$\langle 3, 3\rangle$

16. $x=2\sqrt{3}\cos\left(\frac{180}{\pi}\frac{\pi}{6}\right)=3$

$y=2\sqrt{3}\sin\left(\frac{180}{\pi}\frac{\pi}{6}\right)=\sqrt{3}$

$\langle 3, \sqrt{3}\rangle$

17. (a) $\langle 3(3),\ 3(-2)\rangle=\langle 9, -6\rangle$

(b) $\sqrt{9^2+(-6)^2}=\sqrt{117}=3\sqrt{13}$

18. (a) $\langle -2(-2), -2(5)\rangle=\langle 4, -10\rangle$

(b) $\sqrt{4^2+(-10)^2}=\sqrt{116}=2\sqrt{29}$

19. (a) $\langle 3+(-2), -2+5\rangle=\langle 1, 3\rangle$

(b) $\sqrt{1^2+3^2}=\sqrt{10}$

20. (a) $\langle 3-(-2), -2-5\rangle=\langle 5, -7\rangle$

(b) $\sqrt{5^2+(-7)^2}=\sqrt{74}$

21. (a) $2\mathbf{u}=\langle 2(3), 2(-2)\rangle=\langle 6, -4\rangle$

$3\mathbf{v}=\langle 3\,(-2),\ 3(5)\rangle=\langle -6, 15\rangle$

$2\mathbf{u}-3\mathbf{v}=\langle 6-(-6), -4-15\rangle=\langle 12, -19\rangle$

(b) $\sqrt{12^2+(-19)^2}=\sqrt{505}$

22. (a) $-2\mathbf{u}=\langle -2(3), -2(-2)\rangle=\langle -6, 4\rangle$

$5\mathbf{v}=\langle 5\,(-2),\ 5\,(5)\rangle=\langle -10,\ 25\rangle$

$-2\mathbf{u}+5\mathbf{v}=\langle -6+(-10),\ 4+25\rangle=\langle -16,\ 29\rangle$

(b) $\sqrt{(-16)^2+29^2}=\sqrt{1097}$

23. (a) $\frac{3}{5}\mathbf{u}=\left\langle \frac{3}{5}(3),\ \frac{3}{5}(-2)\right\rangle=\left\langle \frac{9}{5}, -\frac{6}{5}\right\rangle$

$\frac{4}{5}\mathbf{v}=\left\langle \frac{4}{5}(-2),\ \frac{4}{5}(5)\right\rangle=\left\langle -\frac{8}{5},\ 4\right\rangle$

$\frac{3}{5}\mathbf{u}+\frac{4}{5}\mathbf{v}=\left\langle \frac{9}{5}+\left(-\frac{8}{5}\right), -\frac{6}{5}+4\right\rangle=\left\langle \frac{1}{5},\ \frac{14}{5}\right\rangle$

(b) $\sqrt{\left(\frac{1}{5}\right)^2+\left(\frac{14}{5}\right)^2}=\frac{\sqrt{197}}{5}$

24. **(a)** $-\frac{5}{13}\mathbf{u} = \left\langle -\frac{5}{13}(3),\ -\frac{5}{13}(-2)\right\rangle = \left\langle -\frac{15}{13},\ \frac{10}{13}\right\rangle$

$\frac{12}{13}\mathbf{v} = \left\langle \frac{12}{13}(-2),\ \frac{12}{13}(5)\right\rangle = \left\langle -\frac{24}{13},\ \frac{60}{13}\right\rangle$

$-\frac{5}{13}\mathbf{u} + \frac{12}{13}\mathbf{v} = \left\langle -\frac{15}{13} + \left(-\frac{24}{13}\right),\ \frac{10}{13} + \frac{60}{13}\right\rangle = \left\langle -3,\ \frac{70}{13}\right\rangle$

(b) $\sqrt{(-3)^2 + \left(\frac{70}{13}\right)^2} = \frac{\sqrt{6421}}{13}$

25. Initial velocity is 70° north of east:
$325\langle \cos 70°,\ \sin 70°\rangle \approx \langle 111.157,\ 305.400\rangle$.
Wind velocity is 130° north of east:
$40\langle \cos 130°,\ \sin 130°\rangle \approx \langle -25.712,\ 30.642\rangle$.
Add the two vectors to get $\approx \langle 85.445,\ 336.042\rangle$.
The speed is the magnitude, ≈ 346.735 mph.
The direction is $\tan^{-1}\left(\frac{336.042}{85.445}\right) \approx 75.734°$ north of east,
or $\approx 14.266°$ east of north.

26. $x = 4\cos 135 = -2\sqrt{2}$
$y = 4\sin 135 = 2\sqrt{2}$
The true velocity is $\langle 2 - 2\sqrt{2},\ 2\sqrt{2}\rangle$, so the true angle is

$\theta = \cos^{-1}\left(\frac{2 - 2\sqrt{2}}{4}\right) = 106.3°$ and the true speed is

$\sqrt{(2 - 2\sqrt{2})^2 + (2\sqrt{2})^2} = 2.95$ mph

27. $v = \frac{dr(t)}{dt} = \frac{d}{dt}\langle 3t^2,\ 2t^3\rangle = \langle 6t,\ 6t^2\rangle$

$a = \frac{dv(t)}{dt} = \frac{d}{dt}\langle 6t, 6t^2\rangle = \langle 6, 12t\rangle$

28. $v = \frac{dr(t)}{dt} = \frac{d}{dt}\langle \sin 2t, 2\cos t\rangle = \langle 2\cos 2t, -2\sin t\rangle$

$a = \frac{dv(t)}{dt} = \frac{d}{dt}\langle 2\cos 2t, -2\sin t\rangle = \langle -4\sin 2t, -2\cos t\rangle$

29. $v = \frac{dr(t)}{dt} = \frac{d}{dt}\langle te^{-t}, e^{-t}\rangle = \langle e^{-t} - te^{-t}, -e^{-t}\rangle$

$a = \frac{dv(t)}{dt} = \frac{d}{dt}\langle e^{-t} - te^{-t},\ -e^{-t}\rangle = \langle -2e^{-t} + te^{-t},\ e^{-t}\rangle$

30. $v = \frac{dr(t)}{dt} = \frac{d}{dt}\langle 2\cos 3t, 2\sin 4t\rangle = \langle -6\sin 3t, 8\cos 4t\rangle$

$a = \frac{dv(t)}{dt} = \frac{d}{dt}\langle -6\sin 3t, 8\cos 4t\rangle = \langle -18\cos 3t, -32\sin 4t\rangle$

31. $v = \frac{dr(t)}{dt} = \frac{d}{dt}\langle t^2 + \sin 2t, t^2 - \cos 2t\rangle$
$= \langle 2t + 2\cos 2t, 2t + 2\sin 2t\rangle$

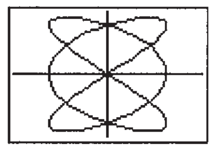
$a = \frac{dv(t)}{dt} = \frac{d}{dt}\langle 2t + 2\cos 2t, 2t + 2\sin 2t\rangle$
$= \langle 2 - 4\sin 2t, 2 + 4\cos 2t\rangle$

32. $v = \frac{dr(t)}{dt} = \frac{d}{dt}\langle t\sin t, t\cos t\rangle = \langle \sin t + t\cos t, \cos t - t\sin t\rangle$

$a = \frac{dv(t)}{dt} = \frac{d}{dt}\langle \sin t + t\cos t, \cos t - t\sin t\rangle$
$= \langle 2\cos t - t\sin t, -2\sin t - t\cos t\rangle$

33. $v = \frac{dr(t)}{dt} = \frac{d}{dt}\langle \cos 3t, \sin 2t\rangle = \langle -3\sin 3t, 2\cos 2t\rangle$

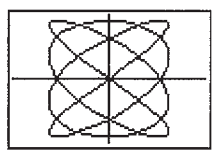
$a = \frac{dv(t)}{dt} = \frac{d}{dt}\langle -3\sin 3t,\ 2\cos 2t\rangle = \langle -9\cos 3t, -4\sin 2t\rangle$



[–1.6, 1.6] by [–1.1, 1.1]
$0 \le t \le 2\pi$

34. $v = \frac{dr(t)}{dt} = \frac{d}{dt}\langle \sin 4t, \cos 3t\rangle = \langle 4\cos 4t, -3\sin 3t\rangle$

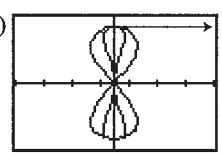
$a = \frac{dv(t)}{dt} = \frac{d}{dt}\langle 4\cos 4t, -3\sin 3t\rangle = \langle -16\sin 4t, -9\cos 3t\rangle$



[–1.6, 1.6] by [–1.1, 1.1]
$0 \le t \le 2\pi$

35. **(a)** $v(t) = \frac{d}{dt}\langle \sin 4t\cos t, \sin 2t\rangle$
$= \langle 4\cos 4t\cos t - \sin t\sin 4t, 2\cos 2t\rangle\big|_{t=(5\pi/4)}$
$= \langle 2\sqrt{2}, 0\rangle$
Speed: $2\sqrt{2}$

(b)



[–4, 4] by [–1.2, 1.2]
$0 \le t \le 2\pi$

(c) To the right

36. **(a)** $v(t) = \frac{d}{dt}\langle e^t + e^{-t},\ e^t - e^{-t}\rangle$
$= \langle e^t - e^{-t},\ e^t + e^{-t}\rangle$

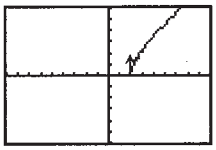
36. Continued

(b) $\lim_{t\to\infty}\dfrac{dy/dt}{dx/dt}=\lim_{t\to\infty}\dfrac{e^t+e^{-t}}{e^t-e^{-t}}=\lim_{t\to\infty}\dfrac{e^{2t}+1}{e^{2t}-1}=1$

(c) For any t,

$$\begin{aligned}x^2-y^2&=(e^t+e^{-t})^2-(e^t-e^{-t})^2\\&=e^{2t}+2+e^{-2t}-(e^{2t}-2+e^{-2t})\\&=4\end{aligned}$$

(d) The velocity at $t=0$ is $\langle 0,\ 2\rangle$.



[−9, 9] by [−6, 6]
$0\le t\le 3$

37. (a) $\left\langle \int_0^3 3t^2-2t\,dt,\ \int_0^3 1+\cos\pi t\,dt\right\rangle+\langle 2,6\rangle$

$$=\left\langle (t^3-t^2)\Big|_0^3,\ \left(t+\frac{1}{\pi}\sin\pi t\right)\Big|_0^3\right\rangle+\langle 2,6\rangle$$

$$=\langle 2+18,6+3\rangle=\langle 20,9\rangle$$

(b) $\int_0^3\sqrt{(3t^2-2t)^2+(1+\cos\pi t)^2}\,dt$

≈ 19.343

38. (a) $\left\langle \int_0^3 2\pi\cos 4\pi t\,dt,\ \int_0^3 4\pi\sin 2\pi t\,dt\right\rangle+\langle 7,2\rangle$

$$=\left\langle \frac{1}{2}\sin 4\pi t\Big|_0^3,\ -2\cos 2\pi t\Big|_0^3\right\rangle+\langle 7,2\rangle$$

$$=\langle 7+0,\ 2+0\rangle=\langle 7,\ 2\rangle$$

(b) $\int_0^3\sqrt{(2\pi\cos 4\pi t)^2+(4\pi\sin 2\pi t)^2}\,dt$

≈ 28.523

39. (a) $\left\langle \int_0^3 (t+1)^{-1}dt,\ \int_0^3 (t+2)^{-2}dt\right\rangle+\langle 3,-2\rangle$

$$=\left\langle \ln(t+1)\Big|_0^3,\ -(t+2)^{-1}\Big|_0^3\right\rangle+\langle 3,-2\rangle$$

$$=\langle 3+\ln 4,\ -1.7\rangle$$

(b) $\int_0^3\sqrt{((t+1)^{-1})^2+((t+2)^{-2})^2}\,dt$

≈ 1.419

40. (a) $\left\langle \int_0^3 e^t-t\,dt,\ \int_0^3 e^t+t\,dt\right\rangle+\langle 1,1\rangle$

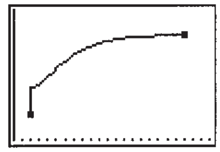
$$=\left\langle \left(e^t-\frac{t^2}{2}\right)\Big|_0^3,\ \left(e^t+\frac{t^2}{2}\right)\Big|_0^3\right\rangle+\langle 1,1\rangle$$

$$=\langle 1+14.586,1+23.586\rangle=\langle 15.586,24.586\rangle$$

(b) $\int_0^3\sqrt{(e^t-t)^2+(e^t+t)^2}\,dt$

≈ 27.791

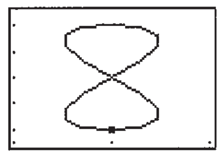
41. The parametric equations are $x=t^3-t^2+2$ and $y=t+\sin(\pi t)/\pi+6$.



[0, 23] by [5, 10]
$0\le t\le 3$

42. The parametric equations are $x=\dfrac{1}{2}\sin(4\pi t)+7$ and $y=-2\cos(2\pi t)+4$.

(Note: The particle traverses the Figure-8 three times, finishing where it started.)



[6, 8] by [1.5, 6.5]
$0\le t\le 3$

43. (a) $v(t)=\dfrac{d}{dt}\left\langle 5\cos\dfrac{\pi}{6}t,3\sin\dfrac{\pi}{6}t\right\rangle$

$$v(t)=\left\langle -\frac{5}{6}\pi\sin\frac{\pi}{6}t,\ \frac{1}{2}\pi\cos\frac{\pi}{6}t\right\rangle$$

$$v(t)=\sqrt{\left(-\frac{5}{6}\pi\sin\frac{\pi}{6}t\right)^2+\left(\frac{1}{2}\pi\cos\frac{\pi}{6}t\right)^2}$$

$$v(2)=\pi\sqrt{\frac{7}{12}}\approx 2.399$$

(b) $a(t)=\dfrac{d}{dt}\left\langle -\dfrac{5}{6}\pi\sin\dfrac{\pi}{6}t,\ \dfrac{1}{2}\pi\cos\dfrac{\pi}{6}t\right\rangle$

$$a(t)=\left\langle -\frac{5\pi^2}{36}\cos\frac{\pi}{6}t,-\frac{\pi^2}{12}\sin\frac{\pi}{6}t\right\rangle\Bigg|_{t=2}$$

$$a(t)=\left\langle -\frac{5\pi^2}{72},-\frac{\pi^2\sqrt{3}}{24}\right\rangle$$

(c) $x^2=\left(5\cos\dfrac{\pi}{6}t\right)^2$

$$y^2=\left(3\sin\frac{\pi}{6}t\right)^2$$

$$\frac{x^2}{25}+\frac{y^2}{9}=1$$

44. (a) $v(t)=\dfrac{d}{dt}\langle\sec\pi t,\tan\pi t\rangle$

$$v(t)=\langle\pi\sec\pi t\tan\pi t,\pi\sec^2\pi t\rangle\big|_{t=1/4}$$

$$v(t)=\langle\sqrt{2}\pi,\ 2\pi\rangle$$

speed: $\sqrt{\left(\sqrt{2}\pi\right)^2+(2\pi)^2}=\sqrt{6}\pi$

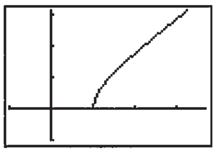
44. Continued

(b) $x^2 = (\sec \pi t)^2$

$y^2 = (\tan \pi t)^2$

$x^2 - y^2 = 1$

(c) The upper part of the right branch:



[−1, 3.7] by [−1, 3.1]
$0 \le t < 1/2$

45. (a) $v(t) = \dfrac{d}{dt}\left\langle \dfrac{1-t^2}{1+t^2}, \dfrac{2t}{1+t^2} \right\rangle$

$v(t) = \left\langle -\dfrac{4t}{(1+t^2)^2}, \dfrac{2-2t^2}{(1+t^2)^2} \right\rangle$

(b) No. The x-component of velocity is zero only if $t = 0$, while the y-component of velocity is zero only if $t = 1$. At no time will the velocity be $\langle 0, 0 \rangle$.

(c) $\lim\limits_{t \to \infty} \left\langle \dfrac{1-t^2}{1+t^2}, \dfrac{2t}{1+t^2} \right\rangle = \langle -1, 0 \rangle$

46. (a) $v(t) = \dfrac{d}{dt}\langle \sin t, \cos 2t \rangle$

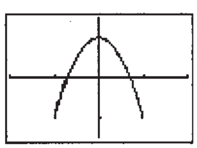
$v(t) = \langle \cos t, -2\sin 2t \rangle$

(b) $v(t) = \langle 0, 0 \rangle$ where $t = \dfrac{\pi}{2}$ and $t = \dfrac{3\pi}{2}$

(c) $\cos 2t = 1 - 2\sin^2 t$

$y = 1 - 2x^2$

(d)



[−2, 2] by [−1.5, 1.5]
$0 \le t \le 2\pi$

47. (a) $\dfrac{d}{dt}\langle e^t \sin t, e^t \cos t \rangle$

$= \langle e^t \sin t + e^t \cos t, e^t \cos t - e^t \sin t \rangle$

$m = \left.\dfrac{e^t \cos t - e^t \sin t}{e^t \sin t + e^t \cos t}\right|_{t=\frac{\pi}{2}} = -1$

(b) $\langle e^1 \sin(1) + e^1 \cos(1), e^1 \cos(1) - e^1 \sin(1) \rangle$

$= \langle 3.756, -0.817 \rangle$

$v(t) = \sqrt{(3.756)^2 + (-0.817)^2} = 3.844$

(c) $\int_0^1 \sqrt{(e^t \sin t + e^t \cos t)^2 + (e^t \cos t - e^t \sin t)^2}\, dt$

≈ 2.430

48. (a) $\dfrac{d}{dt}\left\langle t^2 - 3, \dfrac{2}{3}t^3 \right\rangle$

$= \langle 2t, 2t^2 \rangle$

$|v| = \sqrt{(2(4))^2 + (2(4)^2)^2}$

$|v| = \sqrt{1088} \approx 32.985$

(b) $\int_0^4 ((2t)^2 + (2t^2)^2)^{1/2}\, dt$

≈ 46.062

(c) $t = \sqrt{3+x}$

$y = \dfrac{2}{3}(3+x)^{3/2}$

$\dfrac{dy}{dx} = (3+x)^{1/2}$

49. (a) $3 + \int_2^4 (2 + \sin(t^2))\, dt$

≈ 3.942

(b) $y - y_1 = m(x - x_1)$

$y - 5 = \dfrac{-6}{2 + \sin 4}(x - 3)$

(c) Speed $= \sqrt{(2+\sin 4)^2 + (-6)^2} \approx 6.127$

(d) $\langle 8\cos 16, 2(2+\sin 16) + 7(8)\cos 16 \rangle$

$\approx \langle -7.661, -50.205 \rangle$

50. (a) $y - y_1 = m(x - x_1)$

$y - 5 = \dfrac{3\cos 4}{\sin 8}(x - 4)$

(b) Speed $= \sqrt{(3\cos 4)^2 + (\sin 8)^2} \approx 2.196$

(c) $\int_0^1 \sqrt{(3\cos(t^2))^2 + (\sin(t^3))^2}\, dt$

≈ 2.741

(d) $\left(4 + \int_2^3 \sin(t^3)\, dt, 5 + \int_2^3 3\cos(t^2)\, dt\right)$

$\approx (4.004, 5.724)$

51. False. For example, u and $-1(u)$ have opposite directions.

52. False. For example, $\langle \sqrt{3}, 0 \rangle + \langle 0, 1 \rangle = \langle \sqrt{3}, 1 \rangle$, which has a direction angle of 30°.

53. E. $\dfrac{d}{dt}\langle t^2 + 1, \ln(2t+3) \rangle = \left\langle 2t, \dfrac{1}{2t+3} \right\rangle$

$\dfrac{d}{dt}\left\langle 2t, \dfrac{1}{2t+3} \right\rangle = \left\langle 2, -\dfrac{4}{(2t+3)^2} \right\rangle$

54. D. $\left\langle 4 + \int_0^2 \cos(t^2)\, dt, 7 + \int_0^2 \sin(t^3)\, dt \right\rangle$

$\langle 4 + 0.461, 7 + 0.452 \rangle$

$= \langle 4.461, 7.452 \rangle$

55. B. $x_1 = 7\cos 40 = 5.36$
$y_1 = 7\sin 40 = 4.50$
$x_2 = 4\cos 140 = -3.06$
$y_2 = 4\sin 140 = 2.57$
$\sqrt{(5.36-3.06)^2+(4.50+2.57)^2}$
$= 7.435$

56. B. $\frac{dx}{dt} = 2\cos 2t$
$\frac{dy}{dt} = -5\sin 5t$
Speed $= \sqrt{(2\cos 4)^2+(-5\sin 10)^2} = 3.018$

57. The velocity vector is $\langle -x, \sqrt{1-x^2}\rangle$, which has slope $-\frac{\sqrt{1-x^2}}{x}$. The acceleration vector is

$$\left\langle \frac{d}{dt}(-x), \frac{d}{dt}\left(\sqrt{1-x^2}\right)\right\rangle = \left\langle -\frac{dx}{dt}, \frac{-x}{\sqrt{1-x^2}}\frac{dx}{dt}\right\rangle$$

$= \left\langle x, \frac{x^2}{\sqrt{1-x^2}}\right\rangle$, which has slope $\frac{x}{\sqrt{1-x^2}}$. Since the slopes are negative reciprocals of each other, the vectors are orthogonal.

58. The position vector is $\langle \cos t, \sin t\rangle$, which has slope tan t.
The velocity vector is $\langle -\sin t, \cos t\rangle$, which has slope $-\frac{1}{\tan t}$. The acceleration vector is $\langle -\cos t, -\sin t\rangle$, which has slope tan t.
The velocity slope is the negative reciprocal of the position and acceleration slopes, so velocity is orthogonal to position and to acceleration.

59. **(a)** $t-3 = \frac{3t}{2} - 4$
$t - \frac{3t}{2} = -1$
$t = 2$

Since $t = 2$ also solves $(t-3)^2 = \frac{3t}{2} - 2$, the particles collide when $t = 2$.

(b) First particle: $\mathbf{v}_1(2) = \langle 1, -2\rangle$, so the direction unit vector is $\left\langle \frac{1}{\sqrt{5}}, -\frac{2}{\sqrt{5}}\right\rangle$.

Second particle: $\mathbf{v}_2(t) = \left\langle \frac{3}{2}, \frac{3}{2}\right\rangle$, so the direction unit vector is $\left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\rangle$.

60. **(a)** Referring to the figure, look at the circular arc from the point where $t = 0$ to the point "m". On the one hand, this arc has length given by $r_0\theta$, but it also has length given by vt. Setting these two quantities equal gives the result.

(b) $\mathbf{v}(t) = \left\langle -v\sin\frac{vt}{r_0}, v\cos\frac{vt}{r_0}\right\rangle$ and

$$\mathbf{a}(t) = \left\langle -\frac{v^2}{r_0}\cos\frac{vt}{r_0}, -\frac{v^2}{r_0}\sin\frac{vt}{r_0}\right\rangle = -\frac{v^2}{r_0}\left\langle \cos\frac{v^2}{r_0}, \sin\frac{vt}{r_0}\right\rangle$$

(c) From part (b), $\mathbf{a}(t) = -\left(\frac{v}{r_0}\right)^2 \mathbf{r}(t)$. So, by Newton's second law, $\mathbf{F} = -m\left(\frac{v}{r_0}\right)^2 \mathbf{r}$. Substituting for $\mathbf{F}$ in the law of gravitation gives the result.

(d) Set $\frac{vt}{r_0} = 2\pi$ and solve for vT.

(e) Substitute $\frac{2\pi r_0}{T}$ for v in $v^2 = \frac{GM}{r_0}$ and solve for T^2.

$$\left(\frac{2\pi r_0}{T}\right)^2 = \frac{GM}{r_0}$$
$$\frac{4\pi^2 r_0^{\ 2}}{T^2} = \frac{GM}{r_0}$$
$$\frac{1}{T^2} = \frac{GM}{4\pi^2 r_0^{\ 3}}$$
$$T^2 = \frac{4\pi^2}{GM} r_0^{\ 3}$$

61. Let $\mathbf{u} = \langle a, b\rangle$ be one of the vectors. It has slope $\frac{b}{a}$, so the perpendicular vector $\mathbf{v}$ must have slope $-\frac{a}{b}$. Thus $\mathbf{v} = \langle kb, -ka\rangle$ for some nonzero scalar k, and the dot product is $u \cdot \mathbf{v} = \langle a, b\rangle, \langle kb, -ka\rangle = kab + (-kab) = 0$.

62. **(a)** The diagram shows, by vector addition, that $\mathbf{v} + \mathbf{w} = \mathbf{u}$, so $\mathbf{w} = \mathbf{u} - \mathbf{v}$.

(b) This is just the Law of Cosines applied to the triangle, the sides of which are the magnitudes of the vectors.

(c) By the HMT Rule, $w = \langle u_1 - v_1, u_2 - v_2\rangle$. So

$$\begin{aligned}|\mathbf{u}|^2 + |\mathbf{v}|^2 - |\mathbf{w}|^2 &= \left(u_1^2 + u_2^2\right) + \left(v_1^2 + v_2^2\right) \\ &\quad - \left[\left(u_1 - v_1\right)^2 + \left(u_2 - v_2\right)^2\right] \\ &= u_1^2 + u_2^2 + v_1^2 + v_2^2 \\ &\quad - \left[u_1^2 - 2u_1v_1 + v_1^2 + u_2^2 - 2u_2v_2 + v_2^2\right] \\ &= 2u_1v_1 + 2u_2v_2 \\ &= 2\left(u_1v_1 + u_2v_2\right)\end{aligned}$$

(d) From part (b), $|\mathbf{w}|^2 = |\mathbf{u}|^2 + |\mathbf{v}|^2 - 2|\mathbf{u}||\mathbf{v}|\cos\theta$,
so $|\mathbf{u}|^2 + |\mathbf{v}|^2 - |\mathbf{w}|^2 = 2|\mathbf{u}||\mathbf{v}|\cos\theta$.
From part (c), $|\mathbf{u}|^2 + |\mathbf{v}|^2 - |\mathbf{w}|^2 = 2\left(u_1v_1 + u_2v_2\right)$.
Substituting, we get $2\left(u_1v_1 + u_2v_2\right) = 2|\mathbf{u}||\mathbf{v}|\cos\theta$,
so $\mathbf{u}\cdot\mathbf{v} = u_1v_1 + u_2v_2 = |\mathbf{u}||\mathbf{v}|\cos\theta$.