3. $x^2 = \sin^2 t$
$y^2 = \cos^2 t$
$\cos^2 t + \sin^2 t = 1$
$x^2 + y^2 = 1$

4. $y = \sin 2t = 2\sin t \cos t$
$y = 2x$

5. $x^2 = \tan^2 \theta$
$y^2 = \sec^2 \theta$
$\sec^2 \theta = 1 + \tan^2 x$
$y^2 = 1 + x^2$

6. $x^2 = \csc^2 \theta = 1 + \cot^2 \theta$
$y^2 = \cot^2 \theta$
$x^2 = 1 + y^2$

7. $x^2 = \cos^2 \theta$
$y = \cos 2\theta = 2\cos^2 \theta - 1$
$y = 2x^2 - 1$

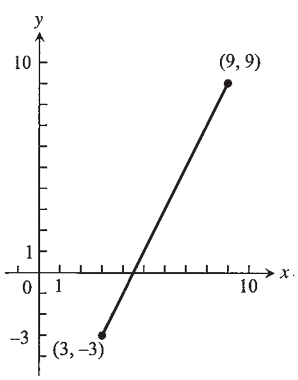
8. $x^2 = \sin^2 \theta$
$y = \cos 2\theta = 1 - 2\sin^2 \theta$
$y = 1 - 2x^2$

9. $x = -\sqrt{\dfrac{y^2}{3} + 1} \qquad (y \geq 0)$

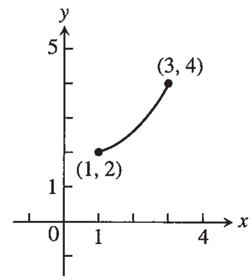
10. $x = \dfrac{1}{e^{y+2}} \qquad (-\infty < y < \infty)$

Section 10.1 Exercises

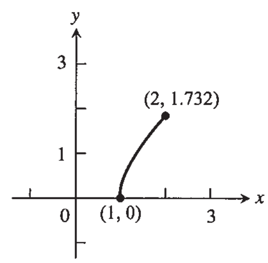
1. Yes, y is a function of x.
$y = 2x - 9$



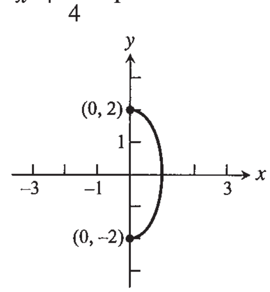
2. Yes, y is a function of x.
$y = \dfrac{x^2 + 7}{4}$



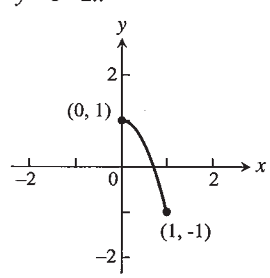
3. Yes, y is a function of x.
$y = \sqrt{x^2 + 1}$



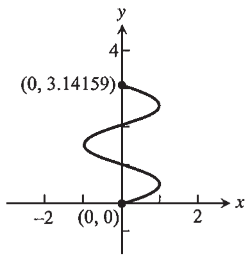
4. No, y is not a function of x.
$x^2 + \dfrac{y^2}{4} = 1$



5. Yes, y is a function of x.
$y = 1 - 2x^2$



6. No, y is not a function of x.
$x = \sin(3y)$



7. (a) $\dfrac{dy}{dx} = \dfrac{-2\sin t}{4\cos t} = -\dfrac{1}{2}\tan t$

(b) $\dfrac{d^2y}{dx^2} = \dfrac{dy'/dt}{dx/dt} = \dfrac{\dfrac{d}{dt}\left(-\dfrac{1}{2}\tan t\right)}{4\cos t} = \dfrac{-\dfrac{1}{2}\sec^2 t}{4\cos t} = -\dfrac{1}{8}\sec^3 t$

8. (a) $\dfrac{dy}{dx} = \dfrac{-\sqrt{3}\sin t}{-\sin t} = \sqrt{3}$

(b) $\dfrac{d^2y}{dx^2} = \dfrac{\dfrac{d}{dt}\left(\sqrt{3}\right)}{-\sin t} = 0$

9. (a) $\dfrac{dy}{dx} = \dfrac{3/2(3t)^{-1/2}}{-(1/2)(t+1)^{-1/2}} = -\sqrt{\dfrac{9t+9}{3t}} = -\sqrt{3+\dfrac{3}{t}}$

(b) $\dfrac{d^2y}{dx^2} = \dfrac{\dfrac{d}{dt}\left(-\sqrt{3+\dfrac{3}{t}}\right)}{-(1/2)(t+1)^{-1/2}} = -\dfrac{\sqrt{3}}{t^{3/2}}$

10. (a) $\dfrac{dy}{dx} = \dfrac{1/t}{-1/t^2} = -t$

(b) $\dfrac{d^2y}{dx^2} = \dfrac{\dfrac{d}{dt}(-t)}{-1/t^2} = t^2$

11. (a) $\dfrac{dy}{dx} = \dfrac{3t^2}{2t-3}$

(b) $\dfrac{d^2y}{dx^2} = \dfrac{\dfrac{d}{dt}\left(\dfrac{3t^2}{2t-3}\right)}{2t-3} = \dfrac{6t^2-18t}{(2t-3)^3}$

12. (a) $\dfrac{dy}{dx} = \dfrac{2t-1}{2t+1}$

(b) $\dfrac{d^2y}{dx^2} = \dfrac{\dfrac{d}{dt}\left(\dfrac{2t-1}{2t+1}\right)}{2t+1} = \dfrac{4}{(2t+1)^3}$

13. (a) $\dfrac{dy}{dx} = \dfrac{\sec t\tan t}{\sec^2 t} = \sin t$

(b) $\dfrac{d^2y}{dx^2} = \dfrac{\dfrac{d}{dt}\sin t}{\sec^2 t} = \cos^3 t$

14. (a) $\dfrac{dy}{dx} = \dfrac{-2\sin(2t)}{-2\sin t} = 2\cos t$

(b) $\dfrac{d^2y}{dx^2} = \dfrac{\dfrac{d}{dt}(2\cos t)}{-2\sin t} = 1$

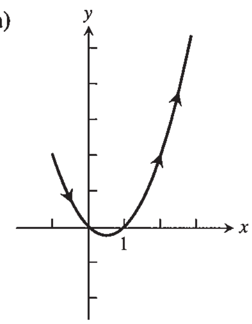
15. (a) $\dfrac{dy}{dx} = \dfrac{4\left(\dfrac{1}{t}\right)}{1/t} = 4$

(b) $\dfrac{d^2y}{dx^2} = \dfrac{\dfrac{d}{dt}(4)}{1/t} = 0$

16. (a) $\dfrac{dy}{dx} = \dfrac{5e^{5t}}{1/t} = 5te^{5t}$

(b) $\dfrac{d^2y}{dx^2} = \dfrac{\dfrac{d}{dt}(5te^{5t})}{1/t} = 25t^2e^{5t} + 5te^{5t}$

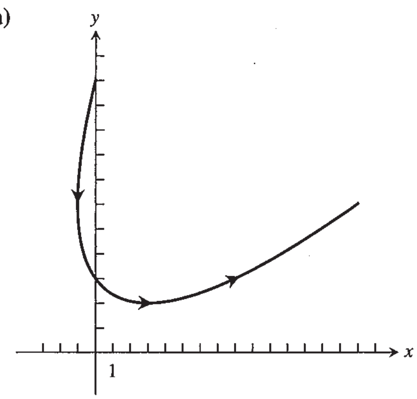
17. (a)



(b) (0.5, –0.25)

(c) We seek to minimize y as a function of t, so we compute $dy/dt = 2t + 1$, which is negative for $-2 \le t < -0.5$ and positive for $-0.5 < t \le 2$. There is a relative minimum at $t = -0.5$, where $(x, y) = (0.5, -0.25)$.

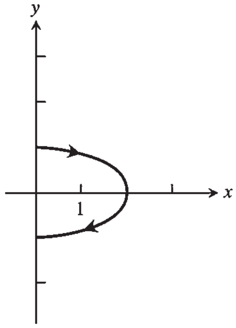
18. (a)



(b) (–1, 6)

(c) We seek to minimize x as a function of t, so we compute $dx/dt = 2t + 2$, which is negative for $-2 \le t < -1$ and positive for $-1 < t \le 3$. There is a relative minimum at $t = -1$, where $(x, y) = (-1, 6)$.

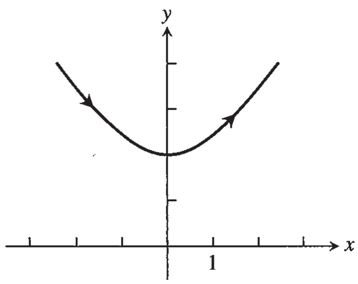
19. (a)



(b) (2, 0)

(c) We seek to maximize x as a function of t, so we compute $dx/dt = 2\cos t$, which is positive for $0 \le t < \pi/2$ and negative for $\pi/2 < t \le \pi$. There is a relative maximum at $t = \pi/2$, where $(x, y) = (2, 0)$.

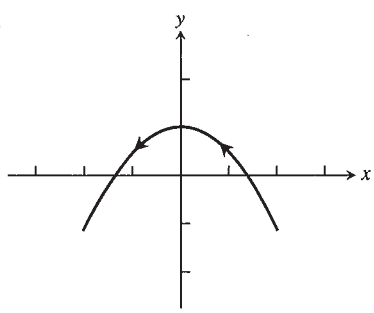
20. (a)



(b) (0, 2)

(c) We seek to minimize y as a function of t, so we compute $dy/dt = 2\sec t\ \tan t$, which is negative for $-1 \le t < 0$ and positive for $0 < t \le 1$. There is a relative minimum at $t = 0$, where $(x, y) = (0, 2)$.

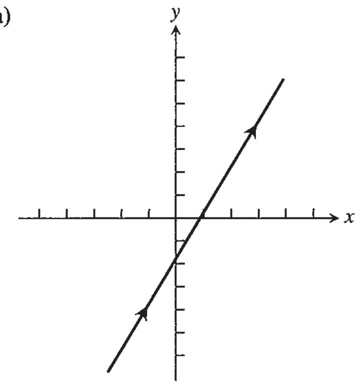
21. (a)



(b) (0, 1)

(c) We seek to maximize y as a function of t, so we compute $dy/dt = -2\sin(2t)$, which is positive for $1.5 \le t < \pi$ and negative for $\pi < t \le 4.5$. There is a relative maximum at $t = \pi$, where $(x, y) = (0, 1)$.

22. (a)



(b) $(\text{In}(50), \text{In}(400)) \approx (3.912, 5.991)$

(c) We seek to maximize x as a function of t, so we compute $dx/dt = 1/t$, which is positive for all $t > 0$. There is an endpoint maximum at $t = 10$, where $(x, y) = (\text{In}(50), \text{In}(400))$.

23. $\dfrac{dy}{dt} = \dfrac{d}{dt}(-1 + \sin t) = \cot t$ and $\dfrac{dx}{dt} = \dfrac{d}{dt}(2 + \cos t)$
$= -\sin t.$

(a) Tangent is horizontal when $\dfrac{dy}{dt} = 0$ and $\dfrac{dx}{dt} \ne 0.$

If $\dfrac{dy}{dt} = 0$, then $\cos t = 0$, and $x = 2 + \cos t = 2$

If $\cos t = 0$, then $\sin t = \pm 1$, and $y = -1 + \sin t = 0$ or 2.

The points are (2, 0) and (2, 2).

(b) Tangent is vertical when $\dfrac{dx}{dt} = 0$ and $\dfrac{dy}{dt} \ne 0.$

If $\dfrac{dx}{dt} = 0$, then $\sin t = 0$, and $y = -1 + \sin t = -1.$

If $\sin t = 0$, then $\cos t = \pm 1$, and $x = 2 + \cos t = 1$ or 3.

The points are (1, –1) and (3, –1).

24. $\dfrac{dy}{dt} = \dfrac{d}{dt}(\tan t) = \sec^2 t$ and $\dfrac{dx}{dt} = \dfrac{d}{dt}(\sec t) = \sec t \tan t.$

(a) Tangent is horizontal when $\dfrac{dy}{dt} = 0$ and $\dfrac{dx}{dt} \ne 0.$

Since $\dfrac{dy}{dt} = \sec^2 t > 0$ for all t, there are no points where the tangent line is horizontal.

(b) Tangent is vertical when $\dfrac{dx}{dt} = 0$ and $\dfrac{dy}{dt} \ne 0.$

If $\dfrac{dx}{dt} = 0$, then $\sec t \tan t = 0$, so $y = \tan t = 0.$

If $\tan t = 0$, then $x = \sec t = \pm 1.$

The points are (1, 0) and (–1, 0).

25. $\frac{dy}{dt}=\frac{d}{dt}(t^3-4t)=3t^2-4$ and $\frac{dx}{dt}=\frac{d}{dt}(2-t)=-1.$

(a) If $\frac{dy}{dt}=0$, then $3t^2-4=0$ and $t=\pm\frac{2}{\sqrt{3}}$. Using the parametric formulas for x and y, the points are $\left(2-2/\sqrt{3},-16\sqrt{3}/9\right)$ and $\left(2+2/\sqrt{3},16\sqrt{3}/9\right)$. (In decimal form, they are (0.845, –3.079) and (3.155, 3.079).)

(b) Tangent is vertical when $\frac{dx}{dt}=0$ and $\frac{dy}{dt}\neq 0$.

Since $\frac{dx}{dt}=-1\neq 0$ for all x, there are no points where the tangent line is vertical.

26. $\frac{dy}{dt}=\frac{d}{dt}(1+3\sin t)=3\cos t$ and $\frac{dx}{dt}=\frac{d}{dt}(-2+3\cos t)$
$=-3\sin t.$

(a) If $\frac{dy}{dt}=0$, then $\cos t=0$ and $x=-2+3\cos t=-2$.

If $\cos t=0$, then $\sin t=\pm 1$, and $y=1+3\sin t=-2$ or 4.
The points are (–2, –2) and (–2, 4).

(b) Tangent is vertical when $\frac{dx}{dt}=0$ and $\frac{dy}{dt}\neq 0$.

If $\frac{dx}{dt}=0$, then $\sin t=0$, and $y=-2+3\cos t=-2$.
If $\sin t=0$, then $\cos t=\pm 1$, and
$x=-2+3\cos t=-5$ or 1.
The points are (–5, 1) and (1, 1).

27. $S=\int_0^{2\pi}\sqrt{(-\sin t)^2+(\cos t)^2}\,dt$

$S=\int_0^{2\pi}\sqrt{1}\,dt$

$S=t\Big|_0^{2\pi}=2\pi-0$

$S=2\pi$

28. $S=\int_0^{\pi}\sqrt{(3\cos t)^2+(-3\sin t)^2}\,dt$

$S=\int_0^{\pi}\sqrt{9}\,dt$

$S=3t\Big|_0^{\pi}=3\pi-0$

$S=3\pi$

29. $S=\int_0^{\pi/2}((-8\sin t+8\sin t+8t\cos t)^2$
$+(8\cos t-8\cos t+8t\sin t)^2)^{1/2}dt$

$S=\int_0^{\pi/2}((8t\cos t)^2+(8t\sin t)^2)^{1/2}dt$

$S=\int_0^{\pi/2}8t\,dt$

$S=4t^2\Big|_0^{\pi/2}=\pi^2-0$

$S=\pi^2$

30. $\int_0^{2\pi}\left(\left[-6\cos^2(t)\sin(t)\right]^2+\left[6\sin^2(t)\cos(t)\right]^2\right)^{1/2}dt$

$=\int_0^{2\pi}6\sqrt{\cos^4(t)\sin^2(t)+\sin^4(t)\cos^2(t)}\,dt$

$=\int_0^{2\pi}6\sqrt{\left\{\cos^2(t)+\sin^2(t)\right\}\left[\cos^2(t)\sin^2(t)\right]}\,dt$

$=\int_0^{2\pi}6\sqrt{\cos^2(t)\sin^2(t)}\,dt$

$=4\bullet\int_0^{\pi/2}6\cos t\sin t\,dt$

$=4\bullet\int_0^{\pi/2}3\bullet\sin 2t\,dt$

$=12\left\{\frac{-\cos 2t}{2}\right\}\Bigg|_0^{\pi/2}=6\bullet[1-(-1)]=12$

31. $S=\int_0^3\left(\left(\sqrt{2t+3}\right)^2+(1+t)^2\right)^{1/2}dt$

$S=\int_0^3(t^2+4t+4)^{1/2}dt$

$S=\frac{21}{2}$

32. $S=\int_0^2\left(\left(\sqrt{8t+8}\right)^2+(2t+1)^2\right)^{1/2}dt$

$S=\int_0^2(4t^2+12t+9)^{1/2}\,dt$

$S=10$

33. $S=\int_0^1((t^2)^2+(t)^2)^{1/2}dt$

$S=\int_0^1(t^4+t^2)^{1/2}\,dt$

$S=\frac{2\sqrt{2}-1}{3}$

34. $S=\int_0^{\pi/3}\sqrt{(\sec t-\cos t)^2+(-\sin t)^2}\,dt$

$=\int_0^{\pi/3}\sqrt{\tan^2 t}\,dt$

$=\int_0^{\pi/3}\tan x\,dx$

$=\left[-\ln|\cos x|\right]_0^{\pi/3}=\ln 2$

35. (a) $x'=-2\sin 2t,\ y'=2\cos 2t,$ so

$\text{Length}=\int_0^{\pi/2}\sqrt{(-2\sin 2t)^2+(2\cos 2t)^2}$

$=\int_0^{\pi/2}2\,dt=\pi.$

(b) $x'=\pi\cos\pi t,\ y'=-\pi\sin\pi t,$ so

$\text{Length}=\int_{-1/2}^{1/2}\sqrt{(\pi\cos\pi t)^2+(-\pi\sin\pi t)^2}\,dt$

$=\int_{-1/2}^{1/2}\pi\,dt=\pi.$

36. $x'=-3\sin t,\ y'=4\cos t,$ so

$\text{Length}=\int_0^{2\pi}\sqrt{(-3\sin t)^2+(4\cos t)^2}\,dt$

which using NINT evaluates to ≈ 22.103.

37. In the first integral, replace t with x. Then $\frac{dx}{dt}$ becomes $\frac{dx}{dx}=1$.

38. Parameterize the curve as $x=g(y), y=y, c\le y\le d$. The parameter is y itself, so replace t with y in the general formula. Then $\frac{dy}{dt}$ becomes $\frac{dy}{dy}=1$.

39. $\frac{dx}{dt}=a(1-\cos t)$

(Note: integrate with respect to x from 0 to $2a\pi$; integrate with respect to t from 0 to 2π.)

$$\begin{aligned}\text{Area} &= \int_0^{2a\pi} y\,dx\\ &= \int_0^{2\pi} a(1-\cos t)a(1-\cos t)\,dt\\ &= a^2\int_0^{2\pi}(1-2\cos t+\cos^2 t)\,dt\\ &= a^2\left[t-2\sin t+\frac{t}{2}+\frac{1}{4}\sin 2t\right]_0^{2\pi}=3\pi a^2\end{aligned}$$

40. $\frac{dx}{dt}=a(1-\cos t)$, so

$$\begin{aligned}\text{Volume} &= \int_0^{2\pi}\pi\left[a(1-\cos t)\right]^2 a(1-\cos t)dt\\ &= \pi a^3\int_0^{2\pi}(1-3\cos t+3\cos^2 t-\cos^3 t)dt\\ &= \pi a^3\left[t-3\sin t+\frac{3}{2}t+\frac{3}{4}\sin 2t-\left(\sin t-\frac{1}{3}\sin^3 t\right)\right]_0^{2\pi}\\ &= 5\pi^2a^3\end{aligned}$$

41. $x=at-b\sin t$ and
$y=a-b\cos t\ (0<b<a)$

42. $x=at-b\sin t$ and
$y=a-b\cos t\ (a<b<2a)$

43. $S=\int_0^{2\pi}((3-2\cos t)^2+(2\sin t)^2)^{1/2}dt$

$S=\int_0^{2\pi}(9-12\cos t+4\cos^2 t+4\sin^2 t)^{1/2}dt$

$S=\int_0^{2\pi}(13-12\cos t)^{1/2}dt$

$S=21.010$

44. $S=\int_0^{2\pi}((2-3\cos t)^2+(3\sin t))^{1/2}dt$

$S=\int_0^{2\pi}(4-12\cos t+9\cos^2 t+9\sin^2 t)^{1/2}dt$

$S=\int_0^{2\pi}(13-12\cos t)^{1/2}dt$

$S=21.010$

45. False. Indeed, y may not even be a function of x. (See Example 1.)

46. True. The ordered pairs $(x, f(x))$ and $(t, f(t))$ are exactly the same.

47. B.

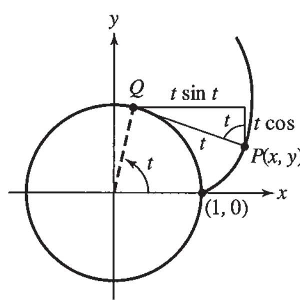
48. C. If $x=\sin t$ and $y=\csc t$, then we can eliminate the parameter to get $y=\frac{1}{x}$. Since sin t and csc t are both positive for $0<t<\frac{\pi}{2}$, the path follows a portion of the curve in the first quadrant, where $y=\frac{1}{x}$ is decreasing and concave up.

49. C. $x=\ln(t)=\ln(y)$

$y=e^x$

50. D.

51. (a) $\overline{QP}$ has length t, so P can be obtained by starting at Q and moving $t\sin t$ units right and $t\cos t$ units downward.
(If either quantity is negative, the corresponding direction is reversed.) Since $Q=(\cos t, \sin t)$, the coordinates of P are
$x=\cos t+t\sin t$ and $y=\sin t-t\cos t$.



(b) $x'=t\cos t,\ y'=t\sin t$, so

$$\begin{aligned}\text{Length} &= \int_0^{2\pi}\sqrt{(t\cos t)^2+(t\sin t)^2}\,dt\\ &= \int_0^{2\pi} t\,dt\\ &= 2\pi^2\end{aligned}$$

52. All distances are a times as big as before.

(a) $x=a(\cot t+t\sin t),\ y=a(\sin t-t\cos t)$

(b) Length $=2a\pi^2$

For exercises 53–56, $x' = v_0 \cos\theta$ and $y' = v_0 \sin\theta - 32t$, and $y = 0$ for $t = 0$ or $t = \frac{v_0 \sin\theta}{16}$. The maximum height is attained in mid-flight at $t = \frac{v_0 \sin\theta}{32}$. To find the path length, evaluate $\int_0^{v_0 \sin\theta/16} \sqrt{(v_0 \cos\theta)^2 + (v_0 \sin\theta - 32t)^2}\,dt$ using NINT. To find the maximum height, calculate

$$y_{\max} = (v_0 \sin\theta)\left(\frac{v_0 \sin\theta}{32}\right) - 16\left(\frac{v_0 \sin\theta}{32}\right)^2.$$

53. **(a)** The projectile hits the ground when $y = 0$.

$$y = t(150 \sin 20° - 16t) = 0$$
$$t = 0 \text{ or } t = \frac{75}{8} \sin 20° \approx 3.206$$
$$x' = 150 \cos 20°,\ y' = 150 \sin 20° - 32t$$

Length =

$$\int_0^{(75\sin 20°)/8} \sqrt{(150 \cos 20°)^2 + (150 \sin 20° - 32t)^2}\,dt$$

which, using NINT, evaluates to ≈ 461.749 ft

(b) The maximum height of the projectile occurs when $y' = 0$,

$$\text{so } t = \frac{75}{16}\sin 20°,\ y\left(\frac{75}{16}\sin 20°\right) \approx 41.125 \text{ ft}$$

54. **(a)** ≈ 641.236 ft

(b) $\frac{5625}{64} \approx 87.891$ ft

55. **(a)** ≈ 840.421 ft

(b) $\frac{16{,}875}{64} \approx 263.672$ ft

56. **(a)** It is not necessary to use NINT.

$$\text{Length} = \int_0^{75/8} (150 - 32t)\,dt = \left[150t - 16t^2\right]_0^{75/8}$$
$$= \frac{5625}{8} = 703.125 \text{ ft}$$

(b) $\frac{5625}{16} = 351.5625$ ft

57. $S = \int_0^{2\pi} 2\pi y\,((-\sin t)^2 + \cos t^2)^{1/2}\,dt$

$$S = \int_0^{2\pi} 2\pi y\,(1)\,dt = \int_0^{2\pi} 2\pi(2 + \sin t)\,dt$$
$$= 2\pi(2t - \cos t)\Big|_0^{2\pi} = 8\pi^2$$

58. $S = \int_0^2 2\pi y\,((t^{-1/2})^2 + (t^{1/2})^2)^{1/2}\,dt$

$$S = \int_0^2 2\pi y\left(\frac{1}{t} + t\right)^{1/2} dt = \int_0^2 2\pi \bullet \frac{2}{3} t^{3/2}\left(\frac{1}{t} + t\right)^{1/2} dt$$
$$= \int_0^2 \frac{4\pi}{3}\left(t^3\left[\frac{1}{t} + t\right]\right)^{1/2} dt = \int_0^2 \frac{4\pi}{3} t(1 + t^2)^{1/2}\,dt$$
$$\approx 14.214$$

59. $S = \int_0^3 2\pi y\,((2t)^2 + (1)^2)^{1/2}\,dt$

$$S = \int_0^3 2\pi y(4t^2 + 1)^{1/2}\,dt$$
$$= \int_0^3 2\pi(t + 1)(4t^2 + 1)^{1/2}\,dt$$
$$= 178.561$$

60. $S = \int_0^{\pi/3} 2\pi y\left((\sec t - \cos t)^2 + (-\sin t)^2\right)^{1/2} dt$

$$S = \int_0^{\pi/3} 2\pi \cos t(\sec^2 t - 1)^{1/2}\,dt$$
$$= \int_0^{\pi/3} 2\pi \cos t \tan t\,dt = 2\pi \int_0^{\pi/3} \sin t\,dt$$
$$= 2\pi(-\cos t)\Big|_0^{\pi/3} = 2\pi\left(-\frac{1}{2} + 1\right) = \pi$$

Section 10.2 Vectors in the Plane (pp. 538–547)

Quick Review 10.2

1. $\sqrt{(5-1)^2 + (3-2)^2} = \sqrt{17}$

2. $\frac{3-2}{5-1} = \frac{1}{4}$

3. Solve $\frac{3-b}{5-3} = \frac{1}{4}$; $b = \frac{5}{2}$.

4. Slope of $\overline{PQ} = -\frac{1}{\overline{RQ}}$, so $\frac{3-2}{5-1} = -\frac{5-3}{3-b}$ and $b = 11$.

5. Slope of $\overline{AB}$ = Slope of $\overline{CD}$, so $\frac{3-0}{1-0} = \frac{3-0}{5-a}$ and $a = 4$.

6. Slope of $\overline{AB}$ = Slope of $\overline{CD}$, so $\frac{5-1}{3-1} = \frac{2-b}{6-8} =$ and $b = 6$.

7. $v(t) = \frac{d}{dt}(t \sin t)$

$$v(t) = \sin t + t \cos t$$
$$a(t) = \frac{d}{dt}(\sin t + t \cos t)$$
$$a(t) = 2\cos t - t \sin t$$

8. $x(t) = \int v(t)dt = \int (3t^2 - 12t)dt$

$$x(t) = t^3 - 6t^2 + C$$
$$x(0) = 0^3 - 6(0)^2 + C = 40$$
$$C = 40$$
$$x(4) = 4^3 - 6(4)^2 + 40 = 8$$

9. $|x(t)| = \left|\int_0^4 v(t)dt\right| = \left|\int_0^4 (3t^2 - 12t)dt\right|$

$$|x(t)| = \left|\left[t^3 - 6t^2\right]_0^4\right| = 32$$

10. $\int_0^{2\pi} \sqrt{(2\cos 2t)^2 + (-3\sin 3t)^2}\,dt$

$$= \int_0^{2\pi} \sqrt{4\cos^2 2t + 9\sin^2 3t}\,dt$$
$$= 15.289$$