Quick Review 6.2

1. $\int_0^2 x^4 dx = \frac{1}{5}x^5\Big]_0^2 = \frac{1}{5}(2)^5 - \frac{1}{5}(0)^5 = \frac{32}{5}$

2. $\int_1^5 \sqrt{x-1}\, dx = \int_1^5 (x-1)^{1/2} dx = \frac{2}{3}(x-1)^{3/2}\Big]_1^5$
$= \frac{2}{3}(4)^{3/2} - \frac{2}{3}(0)^{3/2}$
$= \frac{2}{3}(8) = \frac{16}{3}$

3. $\frac{dy}{dx} = 3^x$

4. $\frac{dy}{dx} = 3^x$

5. $\frac{dy}{dx} = 4(x^3 - 2x^2 + 3)^3(3x^2 - 4x)$

6. $\frac{dy}{dx} = 2\sin(4x-5)\cos(4x-5) \cdot 4$
$= 8\sin(4x-5)\cos(4x-5)$

7. $\frac{dy}{dx} = \frac{1}{\cos x} \cdot -\sin x = -\tan x$

8. $\frac{dy}{dx} = \frac{1}{\sin x} \cdot \cos x = \cot x$

9. $\frac{dy}{dx} = \frac{1}{\sec x + \tan x} \cdot (\sec x \tan x + \sec^2 x)$
$= \frac{\sec x \tan x + \sec^2 x}{\sec x + \tan x}$
$= \frac{\sec x(\tan x + \sec x)}{\sec x + \tan x}$
$= \sec x$

10. $\frac{dy}{dx} = \frac{1}{\csc x + \cot x}(-\csc x \cot x - \csc^2 x)$
$= -\frac{\csc x \cot x + \csc^2 x}{\csc x + \cot x}$
$= -\frac{\csc x(\cot x + \csc x)}{\csc x + \cot x}$
$= -\csc x$

Section 6.2 Exercises

1. $\int(\cos x - 3x^2)\, dx = \sin x - x^3 + C$

2. $\int x^{-2} dx = -x^{-1} + C$

3. $\int\left(t^2 - \frac{1}{t^2}\right) dt = \frac{t^3}{3} + t^{-1} + C$

4. $\int \frac{dt}{t^2+1} = \tan^{-1} t + C$

5. $\int(3x^4 - 2x^{-3} + \sec^2 x)\, dx = \frac{3}{5}x^5 + x^{-2} + \tan x + C$

6. $\int(2e^x + \sec x \tan x - \sqrt{x})\, dx = 2e^x + \sec x - \frac{2}{3}x^{3/2} + C$

7. $(-\cot u + C)' = -(-\csc^2 u) = \csc^2 u$

8. $(-\csc u + C)' = -(-\csc u \cot u) = \csc u \cot u$

9. $\left(\frac{1}{2}e^{2x} + C\right)' = \frac{1}{2}e^{2x}(2) = e^{2x}$

10. $\left(\frac{1}{\ln 5}5^x + C\right)' = \frac{1}{\ln 5}\, 5^x(\ln 5) = 5^x$

11. $(\tan^{-1} u + C)' = \frac{1}{1+u^2}$

12. $(\sin^{-1} u + C)' = \frac{1}{\sqrt{1-u^2}}$

13. $\int f(u)\, du = \int \sqrt{u}\, du = \frac{2}{3}u^{3/2} + C = \frac{2}{3}x^3 + C$
$\int f(u)\, dx = \int \sqrt{u}\, dx = \int \sqrt{x^2} dx = \int x\, dx = \frac{1}{2}x^2 + C$

14. $\int f(u)\, du = \int u^2\, du = \frac{1}{3}u^3 + C = \frac{1}{3}x^{15} + C$
$\int f(u)\, dx = \int u^2\, dx = \int x^{10}\, dx = \frac{1}{11}x^{11} + C$

15. $\int f(u)\, du = \int e^u\, du = e^u + C = e^{7x} + C$
$\int f(u)\, du = \int e^u dx = \int e^{7x} dx = \frac{1}{7}e^{7x} + C$

16. $\int f(u)\, du = \int \sin u\, du = -\cos u + C = -\cos 4x + C$
$\int f(u)\, dx = \int \sin u\, dx = \int \sin 4x\, dx = -\frac{1}{4}\cos 4x + C$

17. $u = 3x$
$du = 3\, dx$
$\frac{1}{3}du = dx$
$\int \sin 3x\, dx = \frac{1}{3}\int \sin u\, du$
$= -\frac{1}{3}\cos u + C$
$= -\frac{1}{3}\cos 3x + C$

Check: $\frac{d}{dx}\left(-\frac{1}{3}\cos 3x + C\right) = -\frac{1}{3}(-\sin 3x)(3) = \sin 3x$

18. $u = 2x^2$

$du = 4x\,dx$

$x\,dx = \frac{1}{4}du$

$$\int x\cos(2x^2)\,dx = \frac{1}{4}\int \cos u\,du = \frac{1}{4}\sin u + C = \frac{1}{4}\sin(2x^2) + C$$

Check: $\frac{d}{dx}\left(\frac{1}{4}\sin(2x^2) + C\right) = \frac{1}{4}\cos(2x^2)(4x) = x\cos(2x^2)$

19. $u = 2x$

$du = 2\,dx$

$\frac{1}{2}du = dx$

$$\int \sec 2x \tan 2x\,dx = \frac{1}{2}\int \sec u \tan u\,du = \frac{1}{2}\sec u + C = \frac{1}{2}\sec 2x + C$$

Check: $\frac{d}{dx}\left(\frac{1}{2}\sec 2x + C\right) = \frac{1}{2}\sec 2x \tan 2x \bullet 2 = \sec 2x \tan 2x$

20. $u = 7x - 2$

$du = 7dx$

$\frac{1}{7}du = dx$

$$\int 28(7x-2)^3 dx = \frac{1}{7}\int 28u^3 du = u^4 + C = (7x-2)^4 + C$$

Check: $\frac{d}{dx}\left[(7x-2)^4 + C\right] = 4(7x-2)^3(7) = 28(7x-2)^3$

21. $u = \frac{x}{3}$ $\qquad x = 3u$

$du = \frac{1}{3}dx$ $\qquad x^2 = 9u^2$

$3\,du = dx$

$$\int \frac{dx}{x^2+9} = \int \frac{3du}{9u^2+9} = \frac{3}{9}\int \frac{du}{u^2+1} = \frac{1}{3}\int \frac{du}{u^2+1} = \frac{1}{3}\tan^{-1} u + C = \frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right) + C$$

Check: $\frac{d}{dx}\left(\frac{1}{3}\tan^{-1}\frac{x}{3} + C\right) = \frac{1}{3}\frac{1}{1+\left(\frac{x}{3}\right)^2} \bullet \frac{1}{3} = \frac{1}{9+x^2}$

22. $u = 1 - r^3$

$du = -3r^2\,dr$

$-\frac{1}{3}du = r^2 dr$

$$\int \frac{9r^2 dr}{\sqrt{1-r^3}} = 9\left(-\frac{1}{3}\right)\int \frac{du}{\sqrt{u}} = -3\int u^{-1/2} du = -3(2)u^{1/2} + C = -6\sqrt{1-r^3} + C$$

Check: $\frac{d}{dx}\left(-6\sqrt{1-r^3} + C\right) = -6\left(\frac{1}{2\sqrt{1-r^3}}\right)(-3r^2) = \frac{9r^2}{\sqrt{1-r^3}}$

23. $u = 1 - \cos\frac{t}{2}$

$du = \frac{1}{2}\sin\frac{t}{2}dt$

$2\,du = \sin\frac{t}{2}dt$

$$\int \left(1-\cos\frac{t}{2}\right)^2 \sin\frac{t}{2}dt = 2\int u^2 du = \frac{2}{3}u^3 + C = \frac{2}{3}\left(1-\cos\frac{t}{2}\right)^3 + C$$

Check: $\frac{d}{dx}\left[\frac{2}{3}\left(1-\cos\frac{t}{2}\right)^3 + C\right]$

$= 2\left(1-\cos\frac{t}{2}\right)^2\left(\sin\frac{t}{2}\right)\left(\frac{1}{2}\right)$

$= \left(1-\cos\frac{t}{2}\right)^2 \sin\frac{t}{2}$

24. $u = y^4 + 4y^2 + 1$

$du = (4y^3 + 8y)\,dy$

$du = 4(y^3 + 2y)\,dy$

$\frac{1}{4}du = (y^3 + 2y)dy$

24. Continued

$$\int 8(y^4+4y^2+1)^2(y^3+2y)dy = 8\left(\frac{1}{4}\right)\int u^2 du$$
$$= \frac{2}{3}u^3 + C$$
$$= \frac{2}{3}(y^4+4y^2+1)^3 + C$$

Check: $\frac{d}{dx}\left[\frac{2}{3}(y^4+4y^2+1)^3+C\right]$

$$= 2(y^4+4y^2+1)^2(4y^3+8y)$$
$$= 8(y^4+4y^2+1)^2(y^3+2y)$$

25. Let $u = 1-x$

$du = -dx$

$$\int \frac{dx}{(1-x)^2} = -\int \frac{du}{u^2}$$
$$= u^{-1} + C$$
$$= \frac{1}{1-x} + C$$

26. Let $u = x+2$

$du = dx$

$$\int \sec^2(x+2)\ dx = \int \sec^2 u\ du$$
$$= \tan u + C$$
$$= \tan(x+2) + C$$

27. Let $u = \tan x$

$du = \sec^2 x\ dx$

$$\int \sqrt{\tan x}\sec^2 x\ dx = \int u^{1/2} du$$
$$= \frac{2}{3}u^{3/2} + C$$
$$= \frac{2}{3}(\tan x)^{3/2} + C$$

28. Let $u = \theta + \frac{\pi}{2}$

$du = d\theta$

$$\int \sec\left(\theta+\frac{\pi}{2}\right)\tan\left(\theta+\frac{\pi}{2}\right)d\theta = \int \sec u \tan u\ du$$
$$= \sec u + C$$
$$= \sec\left(\theta+\frac{\pi}{2}\right) + C$$

29. $\int \tan(4x+2)\ dx$

$u = 4x+2$

$du = 4\ dx$

$\frac{1}{4}du = dx$

$\frac{1}{4}\int \tan u\ du$

$= -\frac{1}{4}\ln|\cos(4x+2)| + C$ or

$\frac{1}{4}\ln|\sec(4x+2)| + C$

30. $\int 3(\sin x)^{-2} dx$

$$= 3\int \frac{1}{\sin^2 x}dx$$
$$= 3\int \csc^2 x\ dx$$
$$= -3\cot x + C$$

31. Let $u = 3z+4$

$du = 3\ dz$

$\frac{1}{3}du = dz$

$$\int \cos(3z+4)dz = \frac{1}{3}\int \cos u\ du$$
$$= \frac{1}{3}\sin u + C$$
$$= \frac{1}{3}\sin(3z+4) + C$$

32. Let $u = \cot x$

$du = -\csc^2 x\ dx$

$$\int \sqrt{\cot x}\csc^2 x\ dx = -\int u^{1/2} du$$
$$= -\frac{2}{3}u^{3/2} + C$$
$$= -\frac{2}{3}(\cot x)^{3/2} + C$$

33. Let $u = \ln x$

$du = \frac{1}{x}dx$

$$\int \frac{\ln^6 x}{x}dx = \int u^6 du$$
$$= \frac{1}{7}u^7 + C$$
$$= \frac{1}{7}(\ln^7 x) + C$$

34. Let $u = \tan\left(\frac{x}{2}\right)$

$du = \frac{1}{2}\sec^2\left(\frac{x}{2}\right)dx$

$$\int \tan^7\left(\frac{x}{2}\right)\sec^2\left(\frac{x}{2}\right)dx = 2\int u^7 du$$
$$= 2\cdot\frac{1}{8}u^8 + C$$
$$= \frac{1}{4}\tan^8\left(\frac{x}{2}\right) + C$$

35. Let $u = s^{4/3} - 8$

$$du = \frac{4}{3}s^{1/3}ds$$

$$\frac{3}{4}du = s^{1/3}ds$$

$$\int s^{1/3}\cos(s^{4/3}-8)\,ds = \frac{3}{4}\int \cos u\,du$$

$$= \frac{3}{4}\sin u + C$$

$$= \frac{3}{4}\sin(s^{4/3}-8) + C$$

36. $\int \frac{dx}{\sin^2 3x} = \int \csc^2 3x\,dx$

Let $u = 3x$

$du = 3\,dx$

$\frac{1}{3}du = dx$

$$\int \csc^2 3x\,dx = \frac{1}{3}\int \csc^2 u\,du$$

$$= -\frac{1}{3}\cot u + C$$

$$= -\frac{1}{3}\cot(3x) + C$$

37. Let $u = \cos(2t+1)$

$du = -\sin(2t+1)(2)dt$

$-\frac{1}{2}du = \sin(2t+1)dt$

$$\int \frac{\sin(2t+1)}{\cos^2(2t+1)}dt = -\frac{1}{2}\int u^{-2}du = \frac{1}{2}u^{-1} + C$$

$$= \frac{1}{2\cos(2t+1)} + C = \frac{1}{2}\sec(2t+1) + C$$

38. Let $u = 2 + \sin t$

$du = \cos t\;dt$

$$\int \frac{6\cos t}{(2+\sin t)^2}dt = 6\int u^{-2}du$$

$$= -6u^{-1} + C$$

$$= -\frac{6}{2+\sin t} + C$$

39. $\int \frac{dx}{x\ln x}$

$u = \ln x$

$du = \frac{dx}{x}$

$x\,du = dx$

$$\int \frac{du}{u} = \ln u = \ln(\ln x) + C$$

40. $\int \tan^2 x\sec^2 x\;dx$

$u = \tan x$

$du = \sec^2 x\;dx$

$\int u^2 du = \frac{1}{3}u^3 + C$

$\frac{1}{3}\tan^3 x + C$

41. $\int \frac{x\,dx}{x^2+1}$

$u = x^2 + 1$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$$\frac{1}{2}\int \frac{du}{x^2+1} = \frac{1}{2}\ln u + C$$

$$= \frac{1}{2}\ln(x^2+1) + C$$

42. Let $u = \frac{x}{5}$ $\qquad 5u = x$

$du = \frac{1}{5}dx \qquad 25u^2 = x^2$

$5du = dx$

$$\int \frac{40\,dx}{x^2+25} = \int \frac{200\,du}{25u^2+5} = \frac{200}{25}\int \frac{du}{u^2+1}$$

$$= 8\tan^{-1}u + C = 8\tan^{-1}\left(\frac{x}{5}\right) + C$$

43. $\int \frac{dx}{\cot 3x} = \int \frac{\sin 3x}{\cos 3x}dx$

Let $u = \cos 3x$

$du = -3\sin 3x\,dx$

$-\frac{1}{3}du = \sin 3x\,dx$

$$\int \frac{dx}{\cot 3x} = -\frac{1}{3}\int \frac{1}{u}du$$

$$= -\frac{1}{3}\ln|u| + C$$

$$= -\frac{1}{3}\ln|\cos 3x| + C$$

(An equivalent expression is $\frac{1}{3}\ln|\sec 3x| + C$.)

44. Let $u = 5x+8$

$du = 5\,dx$

$\frac{1}{5}du = dx$

$$\int \frac{dx}{\sqrt{5x+8}} = \frac{1}{5}\int u^{-1/2}\,du$$
$$= \frac{1}{5} \cdot 2u^{1/2} + C$$
$$= \frac{2}{5}\sqrt{5x+8} + C$$

45. $$\int \sec x\,dx = \int \sec x \cdot \left(\frac{\sec x + \tan x}{\sec x + \tan x}\right)dx$$
$$= \int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x}\,dx$$

Let $u = \sec x + \tan x$

$du = \sec x \tan x + \sec^2 x\,dx$

$$\int \sec x\,dx = \int \frac{1}{u}du = \ln|u| + C = \ln|\sec x + \tan x| + C$$

46. $$\int \csc x\,dx = \int \csc x\left(\frac{\csc x + \cot x}{\csc x + \cot x}\right)dx$$
$$= \int \frac{\csc^2 x + \csc x \cot x}{\csc x + \cot x}\,dx$$

Let $u = \csc x + \cot x$

$du = -\csc x \cot x - \csc^2 x\,dx$

$$\int \csc x\,dx = -\int \frac{1}{u}du$$
$$= -\ln|u| + C$$
$$= -\ln|\csc x + \cot x| + C$$

47. $$\int \sin^3 2x\,dx = \int (\sin^2 2x) \cdot \sin 2x\,dx$$
$$= \int (1-\cos^2 2x) \cdot \sin 2x\,dx$$

Let $u = \cos 2x$

$du = -2\sin 2x\,dx$

$-\frac{1}{2}du = \sin 2x\,dx$

$$= -\frac{1}{2}\int (1-u^2)du = -\frac{1}{2}\left(u - \frac{u^3}{3}\right) + C$$
$$= -\frac{u}{2} + \frac{u^3}{6} + C = -\frac{\cos 2x}{2} + \frac{\cos^3 2x}{6} + C$$

48. $$\int \sec^4 x\,dx = \int (\sec^2 x)\sec^2 x\,dx$$
$$= \int (1+\tan^2 x)\sec^2 x\,dx$$

Let $u = \tan x$

$$du = \sec^2 x\,dx = \int (1+u^2)du$$
$$= u + \frac{u^3}{3} + C = \tan x + \frac{\tan^3 x}{3} + C$$

49. $$\cos 2x = 1 - 2\sin^2 x$$
$$2\sin^2 x = 1 - \cos 2x$$
$$\int 2\sin^2 x\,dx = \int (1-\cos 2x)\,dx$$

Let $u = 2x$

$du = 2dx$

$\frac{1}{2}du = dx$

$$= \frac{1}{2}\int (1-\cos u)\,du = \frac{1}{2}(u - \sin u) + C$$
$$= \frac{1}{2}(2x - \sin 2x) + C = x - \frac{\sin 2x}{2} + C$$

50. $\cos 2x = 2\cos^2 x - 1$

$\cos^2 x = \frac{1}{2}(1+\cos 2x)$

$$\int 4\cos^2 x\,dx = \int 2(1+\cos 2x)\,dx$$

Let $u = 2x$

$du = 2\,dx$

$$= \int (1+\cos u)\,du$$
$$= u + \sin u + C = 2x + \sin 2x + C$$

51. $$\int \tan^4 x\,dx = \int \tan^2 x \cdot \tan^2 x\,dx$$
$$= \int \tan^2 x(\sec^2 x - 1)dx$$
$$= \int (\tan^2 x \sec^2 x - \tan^2 x)dx$$
$$= \int (\tan^2 x \sec^2 x - \sec^2 x + 1)dx$$
$$= \int (\tan^2 x \sec^2 x - \sec^2 x)dx + \int 1\,dx$$
$$= \int (\tan^2 x - 1)\sec^2 x\,dx + \int dx$$

Let $u = \tan x$

$du = \sec^2 x\,dx$

$$= \int (u^2 - 1)\,du + \int dx$$
$$= \frac{1}{3}u^3 - u + x + C$$
$$= \frac{1}{3}\tan^3 x - \tan x + x + C$$

52. $$\int (\cos^4 x - \sin^4 x)dx$$
$$= \int (\cos^2 x + \sin^2 x)(\cos^2 x - \sin^2 x)dx$$
$$= \int (1)(\cos 2x)dx$$
$$= \frac{1}{2}\sin 2x + C$$

53. Let $u = y+1$ $\quad u(0) = 0+1 = 1$

$du = dy$ $\quad u(3) = 3+1 = 4$

$$\int_0^3 \sqrt{y+1}\,dy = \int_1^4 u^{1/2}du$$
$$= \frac{2}{3}u^{3/2}\Big]_1^4 = \frac{2}{3}(4)^{3/2} - \frac{2}{3}(1)^{3/2}$$
$$= \frac{2}{8}(8) - \frac{2}{3} = \frac{14}{3}$$

54. Let $u = 1 - r^2$ $\quad u(0) = 1 - 0^2 = 1$

$du = -2r\,dr$ $\quad u(1) = 1 - 1^2 = 0$

$-\frac{1}{2}du = r\,dr$

$$\int_0^1 r\sqrt{1-r^2}\,dr = -\frac{1}{2}\int_1^0 u^{1/2}\,du = -\frac{1}{3}u^{3/2}\Big|_1^0$$
$$= -\frac{1}{3}0^{3/2} + \frac{1}{3}\cdot 1^{3/2}$$
$$= \frac{1}{3}$$

55. Let $u = \tan x$, $u\left(-\frac{\pi}{4}\right) = \tan\left(-\frac{\pi}{4}\right) = -1$

$u(0) = \tan(0) = 0$

$du = \sec^2 x\,dx$

$$\int_{-\pi/4}^0 \tan x \sec^2 x\,dx = \int_{-1}^0 u\,du$$
$$= \frac{1}{2}u^2\Big]_{-1}^0$$
$$= \frac{1}{2}(0) - \frac{1}{2}(-1)^2$$
$$= -\frac{1}{2}$$

56. Let $u = 4 + r^2$ $\quad u(1) = 4 + 1^2 = 5$

$du = 2r\,dr$ $\quad u(-1) = 4 + (-1)^2 = 5$

$\frac{1}{2}du = r\,dr$

$$\int_{-1}^1 \frac{5r}{(4+r^2)^2}\,dr = \frac{5}{2}\int_5^5 u^{-2}\,du = 0$$

57. Let $u = 1 + \theta^{3/2}$ $\quad u(0) = 1 + 0 = 1$

$du = \frac{3}{2}\theta^{1/2}\,d\theta$ $\quad u(0) = 1 + 1 = 2$

$\frac{2}{3}du = \theta^{1/2}\,d\theta$

$$\int_0^1 \frac{10\sqrt{\theta}}{(1+\theta^{3/2})^2}\,d\theta = \frac{2}{3}(10)\int_1^2 u^{-2}\,du$$
$$= -\frac{20}{3}u^{-1}\Big]_1^2$$
$$= -\frac{20}{3}\left(\frac{1}{2} - 1\right)$$
$$= -\frac{20}{3}\left(-\frac{1}{2}\right)$$
$$= \frac{10}{3}$$

58. Let $u = 4 + 3\sin x$ $\quad u(-\pi) = 4 + 3\sin(-\pi) = 4$

$du = 3\cos x\,dx$ $\quad u(\pi) = 4 + 3\sin\pi = 4$

$\frac{1}{3}du = \cos x\,dx$

$$\int_{-\pi}^{\pi} \frac{\cos x}{\sqrt{4+3\sin x}}\,dx = \frac{1}{3}\int_4^4 u^{-1/2}\,du = 0$$

59. Let $u = t^5 + 2t$ $\quad u(0) = 0 + 0 = 0$

$du = (5t^4 + 2)\,dt$ $\quad u(1) = 1 + 2 = 3$

$$\int_0^1 \sqrt{t^5+2t}\,(5t^4+2)\,dt = \int_0^3 u^{1/2}\,du$$
$$= \frac{2}{3}u^{3/2}\Big]_0^3$$
$$= \frac{2}{3}(3)^{3/2}$$
$$= \frac{2}{3}\sqrt{27} = 2\sqrt{3}$$

60. Let $u = \cos 2\theta$ $\quad u(0) = \cos 0 = 1$

$du = -2\sin 2\theta\,d\theta$ $\quad u\left(\frac{\pi}{6}\right) = \cos\frac{\pi}{3} = \frac{1}{2}$

$-\frac{1}{2}du = \sin 2\theta\,d\theta$

$$\int_0^{\pi/6} \cos^{-3} 2\theta \sin 2\theta\,d\theta = -\frac{1}{2}\int_1^{1/2} u^{-3}\,du$$
$$= -\frac{1}{2}\cdot\left(-\frac{1}{2}\right)u^{-2}\Big]_1^{1/2}$$
$$= \frac{1}{4}\left(\left(\frac{1}{2}\right)^{-2} - 1\right)$$
$$= \frac{1}{4}(3) = \frac{3}{4}$$

61. $\int_0^7 \frac{dx}{x+2}$

$u = x + 2$ $\quad u(0) = 0 + 2 = 2$

$du = dx$ $\quad u(7) = 7 + 2 = 9$

$$\int_2^9 \frac{du}{u} = \ln|u|\Big|_2^9 = \ln\left(\frac{9}{2}\right) = 1.504$$

62. $\int_2^5 \frac{dx}{2x-3}$

$u = 2x - 3$ $\quad u(2) = 4 - 3 = 1$

$du = 2\,dx$ $\quad u(5) = 10 - 3 = 7$

$\frac{1}{2}du = dx$

$$\frac{1}{2}\int_1^7 \frac{du}{u} = \frac{1}{2}\ln|u|\Big|_1^7 = \frac{1}{2}\ln 7 - \frac{1}{2}\ln 1 = \frac{1}{2}\ln 7 = 0.973$$

63. $\int_1^2 \frac{dt}{t-3}$

$u = t-3 \qquad u(1) = 1-3 = -2$

$du = dt \qquad u(2) = 2-3 = -1$

$$\int_{-2}^{-1} \frac{du}{u} = \ln|u|\Big|_{-2}^{-1} = \ln|-1| - \ln|-2|$$
$$= \ln 1 - \ln 2 = \ln\left(\frac{1}{2}\right) = -0.693$$

64. $$\int_{\pi/4}^{3\pi/4} \cot x\, dx = \int_{\pi/4}^{3\pi/4} \frac{\cos x\, dx}{\sin x}$$

$u = \sin x \qquad u\left(\frac{\pi}{4}\right) = \sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

$du = \cos x\, dx \qquad u\left(\frac{3\pi}{4}\right) = \sin\left(\frac{3\pi}{4}\right) = \frac{\sqrt{2}}{2}$

$$\int_{\sqrt{2}/2}^{\sqrt{2}/2} \frac{du}{u} = 0$$

65. $\int_{-1}^{3} \frac{x\, dx}{x^2+1}$

$u = x^2+1 \qquad u(-1) = (-1)^2+1 = 2$

$du = 2x\, dx \qquad u(3) = 3^2+1 = 10$

$\frac{1}{2} du = x\, dx$

$$\frac{1}{2}\int_2^{10} \frac{du}{u} = \frac{1}{2}\ln|u|\Big|_2^{10} = \frac{1}{2}(\ln 10 - \ln 2) = \frac{1}{2}\ln 5 = 0.805$$

66. $\int_0^2 \frac{e^x dx}{3+e^x}$

$u = 3+e^x \qquad u(0) = 3+e^0 = 3+1 = 4$

$du = e^x\, dx \qquad u(2) = 3+e^2$

$$\int_4^{3+e^2} \frac{du}{u} = \ln|u|\Big|_4^{3+e^2} = \ln(3+e^2) - \ln 4 = 0.954$$

67. Let $u = x^4+9$, $du = 4x^3\, dx$.

$u(0) = 0+9 = 9, \qquad u(1) = 1+9 = 10$

(a) $$\int_0^1 \frac{x^3 dx}{\sqrt{x^4+9}} = \int_9^{10} \frac{1}{4}u^{-1/2}\, du = \frac{1}{2}u^{1/2}\Big]_9^{10}$$
$$= \frac{1}{2}\sqrt{10} - \frac{1}{2}\sqrt{9}$$
$$= \frac{1}{2}\sqrt{10} - \frac{3}{2} \approx 0.081$$

(b) $$\int \frac{x^3}{x^4+9} dx = \int \frac{1}{4}u^{-1/2}\, du$$
$$= \frac{1}{2}u^{1/2} + C$$
$$= \frac{1}{2}\sqrt{x^4+9} + C$$

$$\int_0^1 \frac{x^3}{x^4+9}\, dx = \frac{1}{2}\sqrt{x^4+9}\Big]_0^1$$
$$= \frac{1}{2}\sqrt{10} - \frac{1}{2}\sqrt{9}$$
$$= \frac{1}{2}\sqrt{10} - \frac{3}{2} \approx 0.081$$

68. Let $u = 1-\cos 3x$, $du = 3\sin 3x\, dx$.

$u\left(\frac{\pi}{6}\right) = 1 - \cos\frac{\pi}{2} = 1$

$u\left(\frac{\pi}{3}\right) = 1 - \cos\pi = 2$

(a) $$\int_{\pi/6}^{\pi/3} (1-\cos 3x)\sin 3x\, dx = \int_1^2 \frac{1}{3}u\, du = \frac{1}{6}u^2\Big]_1^2$$
$$= \frac{1}{6}(2)^2 - \frac{1}{6}(1)^2 = \frac{1}{2}$$

(b) $$\int (1-\cos 3x)\sin 3x\, dx = \int \frac{1}{3}u\, du$$
$$= \frac{1}{6}u^2 + C$$
$$= \frac{1}{6}(1-\cos 3x)^2 + C$$

$$\int_{\pi/6}^{\pi/3} (1-\cos 3x)\sin 3x\, dx = \frac{1}{6}(1-\cos 3x)^2\Big]_{\pi/6}^{\pi/3}$$
$$= \frac{1}{6}(2)^2 - \frac{1}{6}(1)^2 = \frac{1}{2}$$

69. We show that $f'(x) = \tan x$ and $f(3) = 5$, where

$$f(x) = \ln\left|\frac{\cos 3}{\cos x}\right| + 5.$$

$$f'(x) = \frac{d}{dx}\left(\ln\left|\frac{\cos 3}{\cos x}\right| + 5\right)$$
$$= \frac{d}{dx}(\ln|\cos 3| - \ln|\cos x| + 5)$$
$$= -\frac{d}{dx}\ln|\cos x|$$
$$= -\frac{1}{\cos x}(-\sin x) = \tan x$$

$$f(3) = \ln\left|\frac{\cos 3}{\cos 3}\right| + 5 = \ln 1 + 5 = 5$$

70. We show that $f'(x) = \cot x$ and $f(2) = 6$, where

$$f(x) = \ln\left|\frac{\sin x}{\sin 2}\right| + 6 = \ln|\sin x| - \ln|\sin 2| + 6$$

$$f'(x) = \frac{d}{dx}\left(\ln|\sin x| - \ln|\sin 2| + 6\right)$$
$$= \frac{1}{\sin x} \bullet \cos x - 0 + 0$$
$$= \cot x$$

$$f(2) = \ln\left|\frac{\sin 2}{\sin 2}\right| + 6 = \ln 1 + 6 = 6$$

71. False. The interval of integration should change from $[0, \pi/4]$ to $[0, 1]$, resulting in a different numerical answer.

72. True. Use the substitution $u = f(x)$, $du = f'(x)dx$:

$$\int_a^b \frac{f'(x)dx}{f(x)} = \int_{f(a)}^{f(b)} \frac{du}{u} = \ln|u|\Big|_{f(a)}^{f(b)} = \ln|f(b)| - \ln|f(a)|$$
$$= \ln f(b) - \ln f(a) = \ln\left(\frac{f(b)}{f(a)}\right)$$

73. D.

74. E. $\int_0^2 e^{2x}dx = \left.\frac{e^{2x}}{2}\right|_0^2 = \frac{e^4 - 1}{2}$

75. B. $\int_3^5 f(x-a)\,dx = F(x-a)\big|_3^5 = F(5-a) - F(3-a) = 7$

$$\int_{3-a}^{5-a} f(x)\,dx = F(x)\big|_{3-a}^{5-a} = F(5-a) - F(3-a) = 7$$

76. A. $\frac{d}{dx}\sin x = \cos x$

$$\cos\left(-\frac{\pi}{2}\right) = 0$$
$$\cos(0) = 1$$
$$\cos\left(\frac{\pi}{2}\right) = 0$$

77. (a) Let $u = x + 1$

$du = dx$

$$\int \sqrt{x+1}\,dx = \int u^{1/2}du$$
$$= \frac{2}{3}u^{3/2} + C$$
$$= \frac{2}{3}(x+1)^{3/2} + C$$

Alternatively, $\frac{d}{dx}\left(\frac{2}{3}(x+1)^{3/2} + C\right) = \sqrt{x+1}$.

(b) By Part 1 of the Fundamental Theorem of Calculus, $\frac{dy_1}{dx} = \sqrt{x+1}$ and $\frac{dy_2}{dx} = \sqrt{x+1}$, so both are antiderivatives of $\sqrt{x+1}$.

(c) Using NINT to find the values of y_1 and y_2, we have:

x	0	1	2	3	4
y_1	0	1.219	2.797	4.667	6.787
y_2	–4.667	–3.448	–1.869	0	2.120
$y_1 - y_2$	4.667	4.667	4.667	4.667	4.667

$C = 4\frac{2}{3}$

(d) $C = y_1 - y_2$

$$= \int_0^x \sqrt{x+1}\,dx - \int_3^x \sqrt{x+1}\,dx$$
$$= \int_0^x \sqrt{x+1}\,dx + \int_x^3 \sqrt{x+1}\,dx$$
$$= \int_0^3 \sqrt{x+1}\,dx$$

78. (a) $\frac{d}{dx}[F(x) + C]$ should equal $f(x)$.

(b) The slope field should help you visualize the solution curve $y = F(x)$.

(c) The graphs of $y_1 = F(x)$ and $y_2 = \int_0^x f(t)\,dt$ should differ only by a vertical shift C.

(d) A table of values for $y_1 - y_2$ should show that $y_1 - y_2 = C$ for any value of x in the appropriate domain.

(e) The graph of f should be the same as the graph of NDER of $F(x)$.

(f) First, we need to find $F(x)$. Let $u = x^2 + 1$, $du = 2x\,dx$.

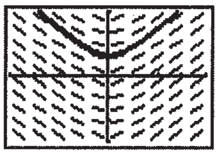
$$\int \frac{x}{\sqrt{x^2+1}}\,dx = \int \frac{1}{2}u^{-1/2}\,du$$
$$= u^{1/2}$$
$$= \sqrt{x^2+1} + C$$

78. Continued

Therefore, we may let $F(x)=\sqrt{x^2+1}$.

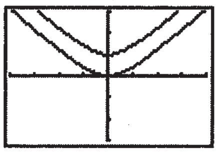
(a) $\frac{d}{dx}(\sqrt{x^2+1}+C)=\frac{1}{2\sqrt{x^2+1}}(2x)$
$=\frac{x}{\sqrt{x^2+1}}$
$=f(x)$

(b)



[−4, 4] by [−3, 3]

(c)

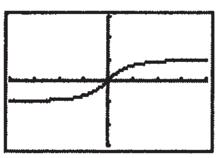


[−4, 4] by [−3, 3]

(d)

x	0	1	2	3	4
y_1	1.000	1.414	2.236	3.162	4.123
y_2	0.000	0.414	1.236	2.162	3.123
$y_1 - y_2$	1	1	1	1	1

(e)



[−4, 4] by [−3, 3]

79. (a) $\int 2\sin x\cos x\,dx=\int 2u\,du=u^2+C=\sin^2 x+C$

(b) $\int 2\sin x\cos x\,dx=-\int 2u\,du=-u^2+C=-\cos^2 x+C$

(c) Since $\sin^2 x-(-\cos^2 x)=1$, the two answers differ by a constant (accounted for in the constant of integration).

80. (a) $\int 2\sec^2 x\tan x\,dx=\int 2u\,du=u^2+C=\tan^2 x+C$

(b) $\int 2\sec^2 x\tan x\,dx=\int 2u\,du=u^2+C=\sec^2 x+C$

(c) Since $\sec^2 x-\tan^2 x=1$, the two answers differ by a constant (accounted for in the constant of integration).

81. (a) $\int\frac{dx}{\sqrt{1-x^2}}=\int\frac{\cos u\,du}{\sqrt{1-\sin^2 u}}$
$=\int\frac{\cos u\,du}{\sqrt{\cos^2 u}}$
$=\int 1du.$

(Note $\cos u>0$, so $\sqrt{\cos^2 u}=|\cos u|=\cos u$.)

(b) $\int\frac{dx}{\sqrt{1-x^2}}=\int 1du$
$=u+C$
$=\sin^{-1}x+C$

82. (a) $\int\frac{dx}{1+x^2}=\int\frac{\sec^2 u\,du}{1+\tan^2 u}$
$=\int\frac{\sec^2 u\,du}{\sec^2 u}$
$=\int 1du$

(b) $\int\frac{dx}{1+x^2}=\int 1du$
$=u+C$
$=\tan^{-1}x+C$

83. (a) $\int_0^{1/2}\frac{\sqrt{x}dx}{\sqrt{1-x}}=\int_{\sin^{-1}\sqrt{0}}^{\sin^{-1}\sqrt{1/2}}\frac{\sin y\cdot 2\sin y\cos y\,dy}{\sqrt{1-\sin^2 y}}$
$=\int_0^{\pi/4}\frac{2\sin^2 y\cos y\,dy}{\cos y}$
$=\int_0^{\pi/4}2\sin^2 y\,dy$

(b) $\int_0^{1/2}\frac{\sqrt{x}dx}{\sqrt{1-x}}=\int_0^{\pi/4}2\sin^2 y\,dy$
$=\int_0^{\pi/4}(1-\cos 2y)dy$
$=[y-(1/2)\sin 2y]_0^{\pi/4}$
$=\left[\frac{\pi}{4}-\frac{1}{2}\sin\frac{\pi}{2}\right]-[0-0]$
$=\frac{(\pi-2)}{4}$

84. (a) $\int_0^{\sqrt{3}} \frac{dx}{\sqrt{1+x^2}} = \int_{\tan^{-1}0}^{\tan^{-1}\sqrt{3}} \frac{\sec^2 u\, du}{\sqrt{1+\tan^2 u}}$

$= \int_0^{\pi/3} \frac{\sec^2 u\, du}{\sec u} = \int_0^{\pi/3} \sec u\, du$

(b) $\int_0^{\sqrt{3}} \frac{dx}{\sqrt{1+x^2}} = \int_0^{\pi/3} \sec u\, du = \left[\ln|\sec u + \tan u|\right]_0^{\pi/3}$

$= \ln\left(2+\sqrt{3}\right) - \ln(1+0) = \ln\left(2+\sqrt{3}\right)$

Section 6.3 Antidifferentiation by Parts (pp. 341–349)

Exploration 1 Choosing the Right *u* and *dv*

1. $u = 1 \qquad du = 0$

$dv = x\cos x \qquad v = \int x\cos x\, dx$

Using 1 for u is never a good idea because it places us back where we started.

2. $u = x\cos x \qquad du = \cos x - x\sin x$

$dv = dx \qquad v = \int dx = 1$

The selection of $u = x\cos x$ will place a more difficult integral into $\int v\,du$.

3. $u = \cos x \quad du = -\sin x$

$dv = x\,dx \qquad v = \int x\,dx = x^2$

The selection of $dv = x\,dx$ will place a more difficult integral into $\int v\,du$.

4. $u = x$ and $dv = \cos x\,dx$ are good choices because the integral is simplified.

Quick Review 6.3

1. $\frac{dy}{dx} = (x^3)(\cos 2x)(2) + (\sin 2x)(3x^2)$

$= 2x^3\cos 2x + 3x^2\sin 2x$

2. $\frac{dy}{dx} = (e^{2x})\left(\frac{3}{3x+1}\right) + \ln(3x+1)(2e^{2x})$

$= \frac{3e^{2x}}{3x+1} + 2e^{2x}\ln(3x+1)$

3. $\frac{dy}{dx} = \frac{1}{1+(2x)^2} \bullet 2$

$= \frac{2}{1+4x^2}$

4. $\frac{dy}{dx} = \frac{1}{\sqrt{1-(x+3)^2}}$

5. $y = \tan^{-1} 3x$

$\tan y = 3x$

$x = \frac{1}{3}\tan y$

6. $y = \cos^{-1}(x+1)$

$\cos y = x+1$

$x = \cos y - 1$

7. $\int_0^1 \sin \pi x\, dx = -\frac{1}{\pi}\cos \pi x\Big]_0^1$

$= -\frac{1}{\pi}\cos\pi + \frac{1}{\pi}\cos 0$

$= -\frac{1}{\pi}(-1) + \frac{1}{\pi} = \frac{2}{\pi}$

8. $\frac{dy}{dx} = e^{2x}$

$dy = e^{2x}dx$

Integrate both sides.

$\int dy = \int e^{2x}dx$

$y = \frac{1}{2}e^{2x} + C$

9. $\frac{dy}{dx} = x + \sin x$

$dy = (x + \sin x)dx$

Integrate both sides.

$\int dy = \int (x+\sin x)dx$

$y = \frac{1}{2}x^2 - \cos x + C$

$y(0) = -1 + C = 2$

$C = 3$

$y = \frac{1}{2}x^2 - \cos x + 3$

10. $\frac{d}{dx}\left(\frac{1}{2}e^x(\sin x - \cos x)\right)$

$= \frac{1}{2}e^x(\cos x + \sin x) + (\sin x - \cos x)\frac{1}{2}e^x$

$= \frac{1}{2}e^x\cos x + \frac{1}{2}e^x\sin x + \frac{1}{2}e^x\sin x - \frac{1}{2}e^x\cos x$

$= e^x\sin x$

Section 6.3 Exercises

1. $\int x\sin x\, dx$

$dv = \sin x\, dx \qquad v = \int \sin x\, dx = -\cos x$

$u = x \qquad du = dx$

$-x\cos x - \int -\cos x\, dx = -x\cos x + \sin x + C$

2. $\int xe^x dx$

$dv = e^x dx \qquad v = \int e^x dx = e^x$

$u = x \qquad du = dx$

$xe^x - \int e^x dx = xe^x - e^x + C$