4. $y' = \frac{1}{2}\cos\frac{x}{2}$

$y'' = -\frac{1}{4}\sin\frac{x}{2}$

$y'' \le 0$ on $[48\pi, 50\pi]$, so the curve is concave down on $[48\pi, 50\pi]$.

5. $y' = 2e^{2x}$

$y'' = 4e^{2x}$

$y'' > 0$ on $[-5, 5]$, so the curve is concave up on $[-5, 5]$.

6. $y' = \frac{1}{x}$

$y'' = -\frac{1}{x^2}$

$y'' < 0$ on $[100, 200]$, so the curve is concave down on $[100, 200]$.

7. $y' = -\frac{1}{x^2}$

$y'' = \frac{2}{x^3}$

$y'' > 0$ on $[3, 6]$, so the curve is concave up on $[3, 6]$.

8. $y' = -\csc x \cot x$

$y'' = (-\csc x)(-\csc^2 x) + (\csc x \cot x)(\cot x)$

$= \csc^3 x + \csc x \cot^2 x$

$y'' > 0$ on $[0, \pi]$, so the curve is concave up on $[0, \pi]$.

9. $y' = -100x^9$

$y'' = -900x^8$

$y'' < 0$ on $[10, 10^{10}]$, so the curve is concave down on $[10, 10^{10}]$.

10. $y' = \cos x + \sin x$

$y'' = -\sin x + \cos x$

$y'' < 0$ on $[1, 2]$, so the curve is concave down.

Section 5.5 Exercises

1. (a) $f(x) = x, h = \frac{2-0}{4} = \frac{1}{2}$

x	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
$f(x)$	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2

$$T = \frac{1}{4}\left(0 + 2\left(\frac{1}{2}\right) + 2(1) + 2\left(\frac{3}{2}\right) + 2\right) = 2$$

(b) $f'(x) = 1, f''(x) = 0$

The approximation is exact.

(c) $\int_0^2 x\,dx = \left[\frac{1}{2}x^2\right]_0^2 = 2$

2. (a) $f(x) = x^2, h = \frac{2-0}{4} = \frac{1}{2}$

x	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
$f(x)$	0	$\frac{1}{4}$	1	$\frac{9}{4}$	4

$$T = \frac{1}{4}\left(0 + 2\left(\frac{1}{4}\right) + 2(1) + 2\left(\frac{9}{4}\right) + 4\right) = 2.75$$

(b) $f'(x) = 2x, f''(x) = 2 > 0$ on $[0,2]$

The aproximation is an overestimate.

(c) $\int_0^2 x^2 dx = \left[\frac{1}{3}x^3\right]_0^2 = \frac{8}{3}$

3. (a) $f(x) = x^3, h = \frac{2-0}{4} = \frac{1}{2}$

x	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
$f(x)$	0	$\frac{1}{8}$	1	$\frac{27}{8}$	8

$$T = \frac{1}{4}\left(0 + 2\left(\frac{1}{8}\right) + 2(1) + 2\left(\frac{27}{8}\right) + 8\right) = 4.25$$

(b) $f'(x) = 3x^2, f''(x) = 6x > 0$ on $[0, 2]$

The aproximation is an overestimate.

(c) $\int_0^2 x^3 dx = \left[\frac{1}{4}x^4\right]_0^2 = 4$

4. (a) $f(x) = \frac{1}{x}, h = \frac{2-1}{4} = \frac{1}{4}$

x	1	$\frac{5}{4}$	$\frac{3}{2}$	$\frac{7}{4}$	2
$f(x)$	1	$\frac{4}{5}$	$\frac{2}{3}$	$\frac{4}{7}$	$\frac{1}{2}$

$$T = \frac{1}{8}\left(1 + 2\left(\frac{4}{5}\right) + 2\left(\frac{2}{3}\right) + 2\left(\frac{4}{7}\right) + \frac{1}{2}\right) \approx 0.697$$

(b) $f'(x) = -\frac{1}{x^2}, f''(x) = \frac{2}{x^3} > 0$ on $[1, 2]$

The approximation is an overestimate.

(c) $\int_1^2 \frac{1}{x} dx = \left[\ln|x|\right]_1^2 = \ln 2 \approx 0.693$

5. (a) $f(x)=\sqrt{x}, h=\frac{4-0}{4}=1$

x	0	1	2	3	4
$f(x)$	0	1	$\sqrt{2}$	$\sqrt{3}$	2

$T=\frac{1}{2}\left(0+2(1)+2\sqrt{2}+2\sqrt{3}+2\right)\approx 5.146$

(b) $f'(x)=-\frac{1}{2}x^{-1/2}, f''(x)=-\frac{1}{4}x^{-3/2}<0$ on $[0, 4]$

The approximation is an underestimate.

(c) $\int_0^4 \sqrt{x}dx=\left[\frac{2}{3}x^{3/2}\right]_0^4=\frac{16}{3}\approx 5.333$

6. (a) $f(x)=\sin x, h=\frac{\pi-0}{4}=\frac{\pi}{4}$

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
$f(x)$	0	$\frac{\sqrt{2}}{2}$	1	$\frac{\sqrt{2}}{2}$	0

$T=\frac{\pi}{8}\left(0+2\left(\frac{\sqrt{2}}{2}\right)+2(1)+2\left(\frac{\sqrt{2}}{2}\right)+0\right)\approx 1.896$

(b) $f'(x)=\cos x, f''(x)=-\sin x<0$ on $[0, \pi]$

The approximation is an underestimate.

(c) $\int_0^\pi \sin x\, dx=\left[-\cos x\right]_0^\pi=2$

7. $T=\frac{h}{2}\left(y_0+2y_1+2y_2+\cdots+2y_{n-1}+y_n\right)$

$\int_0^6 f(x)dx\approx\frac{1}{2}(12+2(10)+2(9)+2(11)+2(13)+2(16)+18)=74$

8. $T=\frac{h}{2}\left(y_0+2y_1+2y_2+\cdots+2y_{n-1}+y_n\right)$

$\int_2^8 f(x)dx\approx\frac{1}{2}(16+2(19)+2(17)+2(14)+2(13)+2(16)+20)=97$

9. $\frac{5}{2}(6.0+2(8.2)+2(9.1)+\cdots+2(12.7)+13.0)(30)$

$=15{,}990\text{ ft}^3$

10. (a) $\frac{200}{2}(0+2(520)+2(800)+2(1000)+\cdots+2(860)$

$+0)(20)=26{,}360{,}000\text{ ft}^3$

(b) You plan to start with 26,360 fish. You intend to have $(0.75)(26{,}360)=19{,}770$ fish to be caught. Since $\frac{19{,}770}{20}=988.5$, the town can sell at most 988 licenses.

11. Sum the trapezoids and multiply by $\frac{1}{3600}$ to change seconds to hours

$\frac{1}{2}(2.0(0+30)+(3.2-2.0)(30+40)+(4.5-3.2)(40+50)$
$+(5.8-4.5)(50+60)+(7.7-5.8)(60+70)$
$+(9.5-7.7)(70+80)+(11.6-9.5)(80+90)$
$+(14.9-11.6)(90+100)+(17.8-14.9)(100+110)$
$+(21.7-17.8)(110+120)+(26.3-21.7)(120+130))$

$\frac{1}{3600}\approx 0.633\text{ mi}\approx 3340\text{ feet}.$

12. Sum the trapezoids and multiply by $\frac{1}{3600}$ to change seconds to hours.

$\frac{1}{3600}\left(\frac{1}{2}\right)(0+2(3)+2(7)+2(12)+2(17)+2(25)+2(33)$
$+2(41)+48)=0.045\text{ mi}\approx 238\text{ feet}.$

13. (a) $\int_0^2 x\,dx=\left(\frac{1/2}{3}\right)\left(0+4\left(\frac{1}{2}\right)+2(1)+4\left(\frac{3}{2}\right)+2\right)=2$

(b) $\int_0^2 x\,dx=\left.\frac{x^2}{2}\right|_0^2=\frac{2^2}{2}+\frac{0^2}{2}=2$

14. (a) $\int_0^2 x^2dx=\left(\frac{1/2}{3}\right)\left(0^2+4\left(\frac{1}{2}\right)^2+2(1)^2+4\left(\frac{3}{2}\right)^2+2^2\right)$

$=\frac{8}{3}$

(b) $\int_0^2 x^2dx=\left.\frac{x^3}{3}\right|_0^2=\frac{2^3}{3}+\frac{0^3}{3}=\frac{8}{3}$

15. (a) $\int_0^2 x^3dx=\left(\frac{1/2}{3}\right)\left(0^3+4\left(\frac{1}{2}\right)^3+2(1)^3+4\left(\frac{3}{2}\right)^3+2^3\right)=4$

(b) $\int_0^2 x^3dx=\left.\frac{x^4}{4}\right|_0^2=\frac{2^4}{4}-\frac{0^4}{4}=4$

16. (a) $\int_1^2 \frac{1}{x}dx=\left(\frac{1/4}{3}\right)\left(\frac{1}{1}+4\left(\frac{1}{1.25}\right)+2\left(\frac{1}{1.5}\right)+4\left(\frac{1}{1.75}\right)+\frac{1}{2}\right)$

≈ 0.69325

(b) $\int_1^2 \frac{1}{x}dx=\ln x|_1^2=\ln 2-\ln 1\approx 0.69315$

17. (a) $\int_0^4 \sqrt{x}\,dx=\left(\frac{1}{3}\right)(\sqrt{0}+4(\sqrt{1})+2(\sqrt{2})+4(\sqrt{3})+(\sqrt{4}))$

≈ 5.2522

(b) $\int_0^4 \sqrt{x}dx=\left.\frac{2}{3}x^{3/2}\right|_0^4=\frac{2}{3}(4)^{3/2}-\frac{2}{3}(0)^{3/2}=\frac{16}{3}$

18. (a) $\int_0^{\pi} \sin x\,dx = \left(\frac{\pi/4}{3}\right)\left(\sin(0)+4\left(\sin\left(\frac{\pi}{4}\right)\right)\right.$
$\left. +2\left(\sin\left(\frac{\pi}{2}\right)\right)+4\left(\sin\left(\frac{3\pi}{4}\right)\right)+\sin\pi\right) \approx 2.00456$

(b) $\int_0^{\pi} \sin x\,dx = -\cos x\Big|_0^{\pi} = -\cos\pi - (-\cos 0) = 2$

19. (a) $f(x) = x^3 - 2x,\ h = \frac{3-(-1)}{4} = 1$

x	-1	0	1	2	3
$f(x)$	1	0	-1	4	21

$S = \frac{1}{3}(1+4(0)+2(-1)+4(4)+21) = 12$

(b) $\int_{-1}^{3}\left(x^3-2x\right)dx = \left[\frac{1}{4}x^4 - x^2\right]_{-1}^{3}$
$= \left(\frac{81}{4}-9\right)-\left(\frac{1}{4}-1\right)$
$= 12$

$|E_s| = 0$

(c) For $f(x) = x^3 - 2x$, $M_{f^{(4)}} = 0$ since $f^{(4)} = 0$.

(d) Simpson's Rule for cubic polynomials will always give exact values since $f^{(4)} = 0$ for all cubic polynomials.

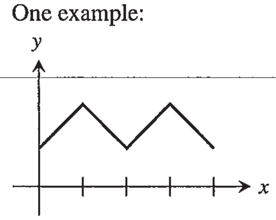
20. The average of the 13 discrete temperatures gives equal weight to the low values at the end.

21. (a) $\frac{1}{2}(126+2\cdot 65+2\cdot 66+\cdots+2\cdot 58+110) = 841$

$av(f) \approx \frac{1}{12}\cdot 841 \approx 70.08$

(b) We are approximating the area under the temperature graph. By doubling the endpoints, the error in the first and last trapezoids increases.

22. Sketch a graph of 4 line segments joined at sharp corners. One example:



23. $S_{50} \approx 3.13791,\ S_{100} \approx 3.14029$

24. $S_{50} \approx 1.08943,\ S_{100} \approx 1.08943$

25. $S_{50} = 1.37066,\ S_{100} = 1.37066$ using $a = 0.0001$ as lower limit

$S_{50} = 1.37076,\ S_{100} = 1.37076$ using $a = 0.000000001$ as lower limit

26. $S_{50} \approx 0.82812,\ S_{100} \approx 0.82812$

27. (a) $T_{10} \approx 1.983523538$

$T_{100} \approx 1.999835504$

$T_{1000} \approx 1.999998355$

(b)

n	$\lvert E_T\rvert = 2 - T_n$
10	$0.016476462 = 1.6476462\times 10^{-2}$
100	1.64496×10^{-4}
1000	1.645×10^{-6}

(c) $\left|E_{T_{10n}}\right| \approx 10^{-2}\left|E_{T_n}\right|$

(d) $b-a=\pi,\ h^2 = \frac{\pi^2}{n^2},\ M = 1$

$\left|E_{T_n}\right| \le \frac{\pi}{12}\left(\frac{\pi^2}{n^2}\right) = \frac{\pi^3}{12n^2}$

$\left|E_{T_{10n}}\right| \le \frac{\pi^3}{12(10n)^2} = 10^{-2}\left|E_{T_n}\right|$

28. (a) $S_{10} \approx 2.000109517$

$S_{100} \approx 2.000000011$

$S_{1000} \approx 2.000000000$

(b)

n	$\lvert E_s\rvert = 2 - S_n$
10	1.09517×10^{-4}
100	1.1×10^{-8}
1000	0

(c) $\left|E_{S_{10n}}\right| = 10^{-4}\left|E_{S_n}\right|$

(d) $b-a=\pi,\ h^4 = \frac{\pi^4}{n^4},\ M = 1$

$\left|E_{S_n}\right| \le \frac{\pi}{180}\left(\frac{\pi^4}{n^4}\right) = \frac{\pi^5}{180n^4}$

$\left|E_{S_{10n}}\right| \le \frac{\pi^5}{180(10n)^4} = 10^{-4}\left|E_{S_n}\right|$

29. $h=\dfrac{24\text{ in.}}{6}=4\text{ in.}$

Estimate the area to be

$$\frac{4}{3}[0+4(18.75)+2(24)+4(26)+2(24)+4(18.75)+0]$$
$$\approx 466.67\,in^2$$

30. Note that the tank cross-section is represented by the shaded area, not the entire wing cross-section. Using Simpson's Rule, estimate the cross-section area to be

$$\frac{1}{3}[y_0+4y_1+2y_2+4y_3+2y_4+4y_5+y_6]$$
$$=\frac{1}{3}[1.5+4(1.6)+2(1.8)+4(1.9)+2(2.0)$$
$$+4(2.1)+2.1]=11.2\text{ ft}^2$$

$$\text{Length}\approx(5000\text{ lb})\left(\frac{1}{42\text{ lb}/\text{ft}^3}\right)\left(\frac{1}{11.2\text{ft}^2}\right)\approx 10.63\text{ ft}$$

31. False. The Trapezoidal Rule will over estimate the integral if it is concave up.

32. False. For example, the two approximations will be the same if f is constant on $[a, b]$.

33. A. LRAM < T < RRAM, so RRAM < 16.4.

34. D. $\displaystyle\int_{-2}^{4}\frac{e^x}{2}dx=\frac{1}{2}\left(2\frac{e^{-2}}{2}+4\frac{e^0}{2}+4\frac{e^2}{2}+2\frac{e^4}{2}\right)$

$$=\frac{1}{2}(e^4+2e^2+2e^0+e^{-2})$$

35. C. $\displaystyle\int_0^{\pi}\sin x\,dx\approx\frac{\pi-0}{2(4)}\left(\sin(0)+2\sin\left(\frac{\pi}{4}\right)\right.$

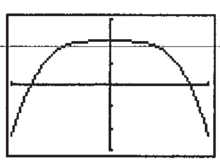
$$\left.+2\sin\left(\frac{\pi}{2}\right)+2\sin\left(\frac{3\pi}{4}\right)+\sin(\pi)\right)$$
$$=\frac{\pi}{8}\left(0+\sqrt{2}+2+\sqrt{2}+0\right)=\frac{\pi}{4}\left(1+\sqrt{2}\right)$$

36. C.

37. **(a)** $f'(x)=2x\cos(x^2)$

$$f''(x)=2x\bullet-2x\sin(x^2)+2\cos(x^2)$$
$$=-4x^2\sin(x^2)+2\cos(x^2)$$

(b)



$[-1, 1]$ by $[-3, 3]$

(c) The graph shows that $-3\le f''(x)\le 2$ so $|f''(x)|\le 3$ for $-1\le x\le 1$.

(d) $|E_T|\le\dfrac{1-(-1)}{12}(h^2)(3)=\dfrac{h^2}{2}$

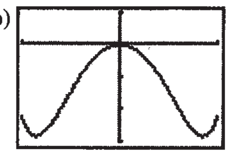
(e) For $0<h\le 0.1, |E_T|\le\dfrac{h^2}{2}\le\dfrac{0.1^2}{2}=0.005<0.01$

(f) $n\ge\dfrac{1-(-1)}{h}\ge\dfrac{2}{0.1}=20$

38. **(a)** $f'''(x)=-4x^2\bullet 2x\cos(x^2)-8x\sin(x^2)-4x\sin(x^2)$

$$=-8x^3\cos(x^2)-12x\sin(x^2)$$
$$f^{(4)}(x)=-8x^3\bullet-2x\sin(x^2)-24x^2\cos(x^2)$$
$$-12x\bullet 2x\cos(x^2)-12\sin(x^2)$$
$$=(16x^4-12)\sin(x^2)-48x^2\cos(x^2)$$

(b)



$[-1, 1]$ by $[-30, 10]$

(c) The graph shows that $-30\le f^{(4)}(x)\le 10$ so $\left|f^{(4)}(x)\right|\le 30$ for $-1\le x\le 1$.

(d) $|E_S|\le\dfrac{1-(-1)}{180}(h^4)(30)=\dfrac{h^4}{3}$

(e) For $0<h\le 0.4, |E_S|\le\dfrac{h^4}{3}\le\dfrac{0.4^4}{3}\approx 0.00853<0.01$

(f) $n\ge\dfrac{1-(-1)}{h}\ge\dfrac{2}{0.4}=5$

39. $T_n=\dfrac{h}{2}[y_0+2y_1+2y_2+\cdots+2y_{n-1}+y_n]$

$$=\frac{h[y_0+y_1+\cdots+y_{n-1}]+h[y_1+y_2+\cdots+y_n]}{2}$$
$$=\frac{\text{LRAM}_n+\text{RRAM}_n}{2}$$

40. $S_{2n}=\dfrac{h}{3}[y_0+4y_1+2y_2+4y_3+\cdots+2y_{2n-2}$

$$+4y_{2n-1}+y_{2n}]$$
$$=\frac{1}{3}[h(y_0+2y_1+2y_2+\cdots+2y_{2n-1}+y_{2n})$$
$$+(2h)(y_1+y_3+y_5+\cdots+y_{2n-1})]$$
$$=\frac{2T_{2n}+\text{MRAM}_n}{3},\text{ where }h=\frac{b-a}{2n}.$$

Quick Quiz Sections 5.4 and 5.5

1. C. $\displaystyle\int_1^7 f(x)\,dx=\frac{1}{2}((4-1)(10+30)+(6-4)(30+40)$

$$+(7-6)(40+20))=160$$

2. D. $\displaystyle\int\sin x^3dx=\left(\frac{-(\sin^2 x)}{3}-\frac{2}{3}\right)\cos x$

$$\left(\frac{-(\sin^2(8))}{3}-\frac{2}{3}\right)\cos 8-\left(\frac{-(\sin^2(1))}{3}-\frac{2}{3}\right)\cos 1=0.632$$