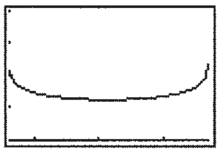
7. We need to choose a closed window narrower than $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$ to avoid the asymptotes.

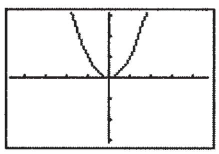


[1.6, 4.7] by [0, 16]

8. The graph of F looks the graph in step 7. It would be decreasing on $\left(\frac{\pi}{2}, \pi\right]$ and increasing on $\left[\pi, \frac{3\pi}{2}\right)$, with vertical asymptotes at $x = \frac{\pi}{2}$ and $x = \frac{3\pi}{2}$.

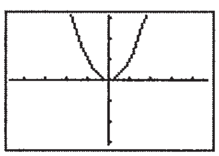
Exploration 2 The Effect of Changing a in $\int_a^x f(t)\,dt$

1.



[−4.7, 4.7] by [−3.1, 3.1]

2.



[−4.7, 4.7] by [−3.1, 3.1]

3. Since NINT $(x^2,\ x, 0, 0) = 0$, the x-intercept is 0.

4. Since NINT $(x^2, x, 5, 5) = 0$, the x-intercept is 5.

5. Changing a has no effect on the graph of $y = \frac{d}{dx}\int_a^x f(t)\,dt$. It will always be the same as the graph of $y = f(x)$.

6. Changing a shifts the graph of $y = \int_a^x f(t)\,dt$ vertically in such a way that a is always the x-intercept. If we change from a_1 to a_2, the distance of the vertical shift is $\int_{a_2}^{a_1} f(t)\,dt$.

Quick Review 5.4

1. $\frac{dy}{dx} = \cos(x^2) \cdot 2x = 2x\cos(x^2)$

2. $\frac{dy}{dx} = 2(\sin x)(\cos x) = 2\sin x \cos x$

3. $\frac{dy}{dx} = 2(\sec x)(\sec x \tan x) - 2(\tan x)(\sec^2 x)$

$= 2\sec^2 x \tan x - 2\tan x \sec^2 x = 0$

4. $\frac{dy}{dx} = \frac{3}{3x} - \frac{7}{7x} = 0$

5. $\frac{dy}{dx} = 2^x \ln 2$

6. $\frac{dy}{dx} = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$

7. $\frac{dy}{dx} = \frac{(-\sin x)(x) - (\cos x)(1)}{x^2} = -\frac{x\sin x + \cos x}{x^2}$

8. $\frac{dy}{dt} = \cos t,\ \frac{dy}{dt} = -\sin t$

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\cos t}{-\sin t} = -\cot t$

9. Implicitly differentiate:

$x\frac{dy}{dx} + (1)y + 1 = 2y\frac{dy}{dx}$

$\frac{dy}{dx}(x - 2y) = -(y+1)$

$\frac{dy}{dx} = -\frac{y+1}{x-2y} = \frac{y+1}{2y-x}$

10. $\frac{dy}{dx} = \frac{1}{dx/dy} = \frac{1}{3x}$

Section 5.4 Exercises

1. $\frac{dy}{dx} = \frac{d}{dx}\int_0^x \left(\sin^2 t\right)dt = \sin^2 x$

2. $\frac{dy}{dx} = \frac{d}{dx}\int_2^x \left(3t + \cos t^2\right)dt = 3x + \cos x^2$

3. $\frac{dy}{dx} = \frac{d}{dx}\int_0^x \left(t^3 - t\right)^5 dt = \left(x^3 - x\right)^5$

4. $\frac{dy}{dx} = \frac{d}{dx}\int_{-2}^x \sqrt{1+e^{5t}}\,dt = \sqrt{1+e^{5x}}$

5. $\frac{dy}{dx} = \frac{d}{dx}\int_0^x \left(\tan^3 u\right)dt = \tan^3 x$

6. $\frac{dy}{dx} = \frac{d}{dx}\int_4^x e^u \sec u\,du = e^x \sec x$

7. $\frac{dy}{dx} = \frac{d}{dx}\int_7^x \frac{1+t}{1+t^2}dt = \frac{1+x}{1+x^2}$

8. $\frac{dy}{dx} = \frac{d}{dx}\int_{-}^x \frac{2-\sin t}{3+\cos t}dt = \frac{2-\sin x}{3+\cos x}$

9. $\frac{dy}{dx} = \frac{d}{dx}\int_0^{x^2} e^{t^2}\,dt = 2x\,e^{x^4}$

10. $\frac{dy}{dx} = \frac{d}{dx}\int_6^{x^2} \cot 3\,t\,dt = \cot x^2 \frac{du}{dx} = 2x \cot 3x^2$

11. $\frac{dy}{dx} = \frac{d}{dx}\int_2^{5x} \frac{\sqrt{1+u^2}}{u}du = \frac{\sqrt{1+25\,x^2}}{x}$

12. $\frac{dy}{dx}=\frac{1+\sin^2(\pi-x)}{1+\cos^2(\pi-x)}\bullet\frac{du}{dx}=\frac{1+\sin^2(\pi-x)}{1+\cos^2(\pi-x)}\bullet(-1)$

13. $\frac{dy}{dx}=\frac{d}{dx}\int_x^6\ln(1+t^2)dt=-\frac{d}{dx}\int_6^x\ln(1+t^2)dt$
$=\ln(1+x^2)(-1)$

14. $\frac{dy}{dx}=\frac{d}{dx}\int_x^7\sqrt{2t^4+t+1}\,dt=-\frac{d}{dx}\int_7^x\sqrt{2t^4+t+1}\,dt$
$=-\sqrt{2x^4+x+1}$

15. $\frac{dy}{dx}=\frac{d}{dx}\int_{x^3}^5\frac{\cos t}{t^2+2}dt=-\frac{d}{dx}\int_5^{x^3}\frac{\cos t}{t^2+2}dt=\frac{\cos x^3}{x^6+2}\frac{du}{dx}$
$=-\frac{3x^2\cos x^3}{x^6+2}$

16. $\frac{dy}{dx}=\frac{d}{dx}\int_{5x^2}^{25}\frac{t^2-2t+9}{t^3+6}dt=-\frac{d}{dx}\int_{25}^{5x^2}\frac{t^2-2t+9}{t^3+6}dt$
$=(-1)\frac{25x^4-10x^2+9}{125x^6+6}\frac{du}{dx}=(-1)\frac{250x^5-100x^3+90x}{125x^6+6}$

17. $\frac{dy}{dx}=\frac{d}{dx}\int_{\sqrt{x}}^0\sin(r^2)\,dr=-\frac{d}{dx}\int_0^{\sqrt{x}}\sin(r^2)\,dr$
$=-\sin x\frac{du}{dx}=-\frac{\sin x}{2\sqrt{x}}$

18. $\frac{dy}{dx}=\frac{d}{dx}\int_{3x^2}^{10}\ln(2+p^2)dp=-\frac{d}{dx}\int_{10}^{3x^2}\ln(2+p^2)dp$
$=-\ln(2+p^2)\frac{du}{dx}=-6x\ln(2+9x^4)$

19. $\frac{dy}{dx}=\frac{d}{dx}\int_{x^2}^{x^3}\cos 2t\,dt=\cos 2x^3\frac{du}{dx}+\cos 2x^2\frac{du}{dx}$
$=3x^2\cos 2x^3-2x\cos 2x^2$

20. $\frac{dy}{dx}=\frac{d}{dx}\int_{\sin x}^{\cos x}t^2dt=\cos^2 x\frac{du}{dx}-\sin^2 x\frac{du}{dx}$
$=-\sin x\cos^2 x-\cos x\sin^2 x$

21. $y=\int_5^x\sin^3 t\,dt$

22. $y=\int_8^x e^t\tan t\,dt$

23. $y=\int_2^x\ln(\sin t+5)dt+3$

24. $y=\int_{-3}^x\sqrt{3-\cos t}\,dt+4$

25. $y=\int_7^x\cos^2 5t\,dt-2$

26. $y=\int_0^x e^{\sqrt{t}}dt+1$

27. $\int_{1/2}^3\left(2-\frac{1}{x}\right)dx=\left[2x-\ln|x|\right]_{1/2}^3$
$=(6-\ln 3)-\left(1-\ln\frac{1}{2}\right)$
$=5-\ln 3+\ln\frac{1}{2}$
$=5-\ln 3-\ln 2$
$=5-\ln 6\approx 3.208$

28. $\int_2^{-1}3^x\,dx=\left[\left(\frac{1}{\ln 3}\right)3^x\right]_2^{-1}$
$=\left(\frac{1}{\ln 3}\right)\left(\frac{1}{3}-9\right)$
$=-\frac{26}{3\ln 3}\approx-7.889$

29. $\int_0^1(x^2+\sqrt{x})\,dx=\left[\frac{1}{3}x^3+\frac{2}{3}x^{3/2}\right]_0^1=\left(\frac{1}{3}+\frac{2}{3}\right)-(0+0)=1$

30. $\int_0^5 x^{3/2}\,dx=\left[\frac{2}{5}x^{5/2}\right]_0^5=\frac{2}{5}(25\sqrt{5}-0)=10\sqrt{5}\approx 22.361$

31. $\int_1^{32}x^{-6/5}dx=\left[-5x^{-1/5}\right]_1^{32}=-5\left(\frac{1}{2}-1\right)=\frac{5}{2}$

32. $\int_{-2}^{-1}\frac{2}{x^2}dx=2\int_{-2}^{-1}x^{-2}dx=2\left[-x^{-1}\right]_{-2}^{-1}=2\left[1-\frac{1}{2}\right]=1$

33. $\int_0^\pi\sin x\,dx=\left[-\cos x\right]_0^\pi=1-(-1)=2$

34. $\int_0^\pi(1+\cos x)\,dx=\left[x+\sin x\right]_0^\pi$
$=(\pi+0)-(0+0)$
$=\pi\approx 3.142$

35. $\int_0^{\pi/3}2\sec^2\theta\,d\theta=2\left[\tan\theta\right]_0^{\pi/3}$
$=2(\sqrt{3}-0)$
$=2\sqrt{3}\approx 3.464$

36. $\int_{\pi/6}^{5\pi/6}\csc^2\theta\,d\theta=\left[-\cot\theta\right]_{\pi/6}^{5\pi/6}$
$=\sqrt{3}-(-\sqrt{3})$
$=2\sqrt{3}\approx 3.464$

37. $\int_{\pi/4}^{3\pi/4}\csc x\cot x\,dx=\left[-\csc x\right]_{\pi/4}^{3\pi/4}=(-\sqrt{2})-(-\sqrt{2})=0$

38. $\int_0^{\pi/3}4\sec x\tan x\,dx=4\left[\sec x\right]_0^{\pi/3}=4(2-1)=4$

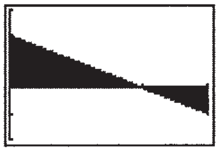
39. $\int_{-1}^1(r+1)^2\,dr=\left[\frac{1}{3}(r+1)^3\right]_{-1}^1=\frac{8}{3}-0=\frac{8}{3}$

40. $\int_0^4 \frac{1-\sqrt{u}}{\sqrt{u}}\,du = \int_0^4 \left(u^{-1/2}-1\right)du$

$= \left[2u^{1/2}-u\right]_0^4$

$= (4-4)-(0-0)=0$

41. Graph $y = 2 - x$.



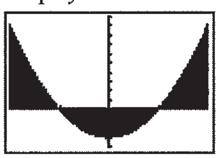
[0, 3] by [–2, 3]

Over [0, 2]: $\int_0^2 (2-x)\,dx = \left[2x-\frac{1}{2}x^2\right]_0^2 = 2$

Over [2, 3]: $\int_2^3 (2-x)\,dx = \left[2x-\frac{1}{2}x^2\right]_2^3 = \frac{3}{2}-2=-\frac{1}{2}$

Total area $= |2|+\left|-\frac{1}{2}\right| = \frac{5}{2}$

42. Graph $y = 3x^2 - 3$.



[–2, 2] by [–4, 10]

Over [–2, –1]:

$\int_{-2}^{-1}\left(3x^2-3\right)dx = \left[x^3-3x\right]_{-2}^{-1} = 2-(-2)=4$

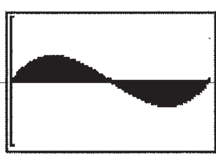
Over [–1, 1]:

$\int_{-1}^{1}\left(3x^2-3\right)dx = \left[x^3-3x\right]_{-1}^{1} = -2-2=-4$

Over [1, 2]: $\int_1^2\left(3x^2-3\right)dx = \left[x^3-3x\right]_1^2 = 2-(-2)=4$

Total area $= |4|+|-4|+|4| = 12$

43. Graph $y = x^3 - 3x^2 - 2x$.



[0, 2] by [–1, 1]

Over [0, 1]:

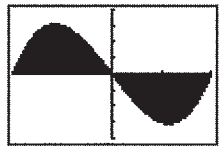
$\int_0^1\left(x^3-3x^2+2x\right)dx = \left[\frac{1}{4}x^4-x^3+x^2\right]_0^1 = \frac{1}{4}-0=\frac{1}{4}$

Over [1, 2]:

$\int_1^2\left(x^3-3x^2+2x\right)dx = \left[\frac{1}{4}x^4-x^3+x^2\right]_1^2 = 0-\frac{1}{4}=-\frac{1}{4}$

Total area $= \left|\frac{1}{4}\right|+\left|-\frac{1}{4}\right| = \frac{1}{2}$

44. Graph $y = x^3 - 4x$.



[–2, 2] by [–4, 4]

Over [–2, 0]:

$\int_{-2}^{0}\left(x^3-4x\right)dx = \left[\frac{1}{4}x^4-2x^2\right]_{-2}^{0} = 0-(-4)=4$

Over [0, 2]:

$\int_0^2\left(x^3-4x\right)dx = \left[\frac{1}{4}x^4-2x^2\right]_0^2 = -4-0=-4$

Total area $= |4|+|-4| = 8$

45. First, find the area under the graph of $y = x^2$.

$\int_0^1 x^2\,dx = \left[\frac{1}{3}x^3\right]_0^1 = \frac{1}{3}$

Next find the area under the graph of $y = 2 - x$.

$\int_1^2 (2-x)\,dx = \left[2x-\frac{1}{2}x^2\right]_1^2 = 2-\frac{3}{2}=\frac{1}{2}$

Area of the shaded region $= \frac{1}{3}+\frac{1}{2}=\frac{5}{6}$

46. First find the area under the graph of $y = \sqrt{x}$.

$\int_0^1 x^{1/2}dx = \left[\frac{2}{3}x^{3/2}\right]_0^1 = \frac{2}{3}$

Next find the area under the graph of $y = x^2$.

$\int_1^2 x^2\,dx = \left[\frac{1}{3}x^3\right]_1^2 = \frac{8}{3}-\frac{1}{3}=\frac{7}{3}$

Area of the shaded region $= \frac{2}{3}+\frac{7}{3}=3$

47. First, find the area under the graph of $y = 1 + \cos x$.

$\int_0^\pi (1+\cos x)\,dx = \left[x+\sin x\right]_0^\pi = \pi$

The area of the rectangle is 2π.

Area of the shaded region $= 2\pi - \pi = \pi$.

48. First, find the area of the region between $y = \sin x$ and the x-axis for $\left[\frac{\pi}{6}, \frac{5\pi}{6}\right]$.

$\int_{\pi/6}^{5\pi/6} \sin x\,dx = \left[-\cos x\right]_{\pi/6}^{5\pi/6} = \frac{\sqrt{3}}{2}-\left(-\frac{\sqrt{3}}{2}\right)=\sqrt{3}$

The area of the rectangle is $\left(\sin\frac{\pi}{6}\right)\left(\frac{2\pi}{3}\right)=\frac{\pi}{3}$

Area of the shaded region $= \sqrt{3}-\frac{\pi}{3}$

49. $\text{NINT}\left(\frac{1}{3+2\sin x}, x, 0, 10\right) \approx 3.802$

50. $\text{NINT}\left(\dfrac{2x^4-1}{x^4-1},\ x,\ -0.8,\ 0.8\right) \approx 1.427$

51. $\dfrac{1}{2}\text{NINT}\left(\sqrt{\cos x},\ x, -1, 1\right) \approx 0.914$

52. $\sqrt{8-2x^2} \ge 0$ between $x = -2$ and $x = 2$

$\text{NINT}(\sqrt{8-2x^2}, x, -2, 2) \approx 8.886$

53. Plot $y_1 = \text{NINT}\left(e^{-t^2},\ t,\ 0,\ x\right), y_2 = 0.6$ in a [0, 1] by [0, 1] window, then use the intersect function to find $x \approx 0.699$.

54. When $y = 0$, $x = 1$.

$y^3 = 1 - x^3$

$y = \sqrt[3]{1-x^3}$

$\text{NINT}(\sqrt[3]{1-x^3},\ x,\ 0,\ 1) \approx 0.883$

55. $\int_a^x f(t)\,dt + K = \int_b^x f(t)\,dt$

$K = -\int_a^x f(t)\,dt + \int_b^x f(t)\,dt$

$= \int_x^a f(t)\,dt + \int_b^x f(t)\,dt$

$= \int_b^a f(t)\,dt$

$K = \int_2^{-1} (t^2 - 3t + 1)\,dt$

$= \left[\dfrac{1}{3}t^3 - \dfrac{3}{2}t^2 + t\right]_2^{-1}$

$= \left[-\dfrac{1}{3} - \dfrac{3}{2} + (-1)\right] - \left[\dfrac{8}{3} - 6 + 2\right] = -\dfrac{3}{2}$

56. To find an antiderivative of $\sin^2 x$, recall from trigonometry that $\cos 2x = 1 - 2\sin^2 x$, so $\sin^2 x = \dfrac{1}{2} - \dfrac{1}{2}\cos 2x$.

$K = \int_2^0 \sin^2 t\,dt$

$= \int_2^0 \left[\dfrac{1}{2} - \dfrac{1}{2}\cos(2x)\right] dx$

$= \left[\dfrac{1}{2}x - \dfrac{1}{4}\sin(2x)\right]_2^0$

$= \left[\dfrac{1}{2}x - \dfrac{1}{2}\sin x \cos x\right]_2^0$

$= 0 - \left(1 - \dfrac{\sin 2 \cos 2}{2}\right) = \dfrac{\sin 2 \cos 2 - 2}{2} \approx -1.189$

57. (a) $H(0) = \int_0^0 f(t)\,dt = 0$

(b) $H'(x) = \dfrac{d}{dx}\left(\int_0^x f(t)\,dt\right) = f(x)$

$H'(x) > 0$ when $f(x) > 0$.

H is increasing on $[0, 6]$.

(c) H is concave up on the open interval where $H''(x) = f'(x) > 0$.

$f'(x) > 0$ when $9 < x \le 12$.

H is concave up on $(9, 12)$.

(d) $H(12) = \int_0^{12} f(t)\,dt > 0$ because there is more area above the x-axis than below the x-axis.

$H(12)$ is positive.

(e) $H'(x) = f(x) = 0$ at $x = 6$ and $x = 12$. Since $H'(x) = f(x) > 0$ on $[0, 6)$, the values of H are increasing to the left of $x = 6$, and since $H'(x) = f(x) < 0$ on $(6, 12]$, the values of H are decreasing to the right of $x = 6$. H achieves its maximum value at $x = 6$.

(f) $H(x) > 0$ on $(0, 12]$. Since $H(0) = 0$, H achieves its minimum value at $x = 0$.

58. (a) $s'(t) = f(t)$. The velocity at $t = 5$ *is* $f(5) = 2$ units/sec.

(b) $s''(t) = f'(t) < 0$ at $t = 5$ since the graph is decreasing, so acceleration at $t = 5$ is negative.

(c) $s(3) = \int_0^3 f(x)\,dx = \dfrac{1}{2}(3)(3) = 4.5$ units

(d) s has its largest value at $t = 6$ sec since $s'(6) = f(6) = 0$ and $s''(6) = f'(6) < 0$.

(e) The acceleration is zero when $s''(t) = f'(t) = 0$. This occurs when $t = 4$ sec and $t = 7$sec.

(f) Since $s(0) = 0$ and $s'(t) = f(t) > 0$ on $(0, 6)$, the particle moves away from the origin in the positive direction on $(0,6)$. The particle then moves in the negative direction, towards the origin, on $(6, 9)$ since $s'(t) = f(t) < 0$ on $(6, 9)$ and the area below the x-axis is smaller than the area above the x-axis.

(g) The particle is on the positive side since $s(9) = \int_0^9 f(x)\,dx > 0$ (the area below the x-axis is smaller than the area above the x-axis).

59. (a) $s'(3) = f(3) = 0$ units / sec

(b) $s''(3) = f'(3) > 0$ so acceleration is positive.

(c) $s(3) = \int_0^3 f(x)\,dx = \dfrac{1}{2}(-6)(3) = -9$ units

(d) $s(6) = \int_0^6 f(x)\,dx = \dfrac{1}{2}(-6)(3) + \dfrac{1}{2}(6)(3) = 0$, so the particle passes through the origin at $t = 6$ sec.

(e) $s''(t) = f'(t) = 0$ at $t = 7$ sec

(f) The particle is moving away from the origin in the negative direction on $(0,3)$ since $s(0) = 0$ and $s'(t) < 0$ on $(0, 3)$. The particle is moving toward the origin on $(3, 6)$ since $s'(t) > 0$ on $(3, 6)$ and $s(6) = 0$. The particle moves away from the origin in the positive direction for $t > 6$ since $s'(t) > 0$.

59. Continued

(g) The particle is on the positive side since $s(9)=\int_0^9 f(x)\,dx>0$ (the area below the x-axis is smaller than the area above the x-axis).

60. $f(x)=\dfrac{d}{dx}\left(\int_1^x f(t)\,dt\right)=\dfrac{d}{dx}\left(x^2-2x+1\right)=2x-2$

61. $f'(x)=\dfrac{d}{dx}\left(2+\int_0^x \dfrac{10}{1+t}\,dt\right)=\dfrac{10}{1+x}$

$f'(0)=10$

$f(0)=2+\int_0^0 \dfrac{10}{1+t}\,dt=2$

$L(x)=2+10x$

62. $f(x)=\dfrac{d}{dx}\left(\int_0^x f(t)\,dt\right)$

$=\dfrac{d}{dx}\left(x\cos\pi x\right)$

$=x\left(-\pi\sin\pi x\right)+1\bullet\cos\pi x$

$=-\pi x\sin\pi x+\cos\pi x$

$f(4)=-4\pi\sin 4\pi+\cos 4\pi=1$

63. One arch of $\sin kx$ is from $x=0$ to $x=\dfrac{\pi}{k}$.

Area $=\int_0^{\pi/k}\sin kx\,dx=\left[-\dfrac{1}{k}\cos kx\right]_0^{\pi/k}=\dfrac{1}{k}-\left(-\dfrac{1}{k}\right)=\dfrac{2}{k}$

64. (a) $\int_{-3}^2\left(6-x-x^2\right)dx=\left[6x-\dfrac{1}{2}x^2-\dfrac{1}{3}x^3\right]_{-3}^2$

$=\dfrac{22}{3}-\left(-\dfrac{27}{2}\right)$

$=\dfrac{125}{6}$

(b) The vertex is at $x=\dfrac{-(-1)}{2(-1)}=-\dfrac{1}{2}$. (Recall that the vertex of a parabola $y=ax^2+bx+c$ is at $x=-\dfrac{b}{2a}$.)

$y\left(-\dfrac{1}{2}\right)=\dfrac{25}{4}$, so the height is $\dfrac{25}{4}$.

(c) The base is $2-(-3)=5$.

$\dfrac{2}{3}(\text{base})(\text{height})=\dfrac{2}{3}(5)\left(\dfrac{25}{4}\right)=\dfrac{125}{6}$

65. True. The Fundamental Theorem of Calculus guarantees that F is differentiable on I, so it must be continuous on I.

66. False. In fact, $\int_a^b e^{x^2}\,dx$ is a real number, so its derivative is always 0.

67. D.

68. D. See the Fundamental Theorem of Calculus.

69. E. $f(a)+f'(a)(x-\pi)$

$f(\pi)=0$

$f'(\pi)=-1$

$y=-1(x-\pi)=\pi-x$

70. E.

71. (a) $f(t)$ is an even function so $\int_{-x}^0 \dfrac{\sin(t)}{t}\,dt=\int_0^x\dfrac{\sin(t)}{t}\,dt.$

$\mathrm{Si}(-x)=\int_0^{-x}\dfrac{\sin(t)}{t}\,dt$

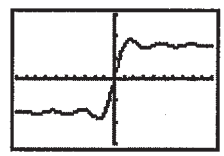
$=-\int_{-x}^0\dfrac{\sin(t)}{t}\,dt$

$=-\int_0^x\dfrac{\sin(t)}{t}\,dt=-\mathrm{Si}(x)$

(b) $\mathrm{Si}(0)=\int_0^0\dfrac{\sin t}{t}\,dt=0$

(c) $\mathrm{Si}'(x)=f(t)=0$ when $t=\pi k$, k a nonzero integer.

(d)



$[-20, 20]$ by $[-3, 20, 203]$

72. (a) $c(100)-c(1)=\int_1^{100}\left(\dfrac{dc}{dx}\right)dx$

$=\int_1^{100}\dfrac{1}{2\sqrt{x}}\,dx=\left[\sqrt{x}\right]_1^{100}$

$=10-1=9$ or \$9

(b) $c(400)-c(100)=\int_{100}^{400}\left(\dfrac{dc}{dx}\right)dx$

$=\int_{100}^{400}\dfrac{1}{2\sqrt{x}}\,dx=\left[\sqrt{x}\right]_{100}^{400}$

$=20-10=10$ or \$10

73. $\int_0^3\left(2-\dfrac{2}{(x+1)^2}\right)dx=\left[2x+2(x+1)^{-1}\right]_0^3$

$=\left[6+\dfrac{1}{2}\right]-2=\dfrac{9}{2}$

$=4.5$ thousand

The company should expect \$4500.

74. (a) $\dfrac{1}{30-0}\int_0^{30}\left(450-\dfrac{x^2}{2}\right)dx=\dfrac{1}{30}\left[450x-\dfrac{x^3}{6}\right]_0^{30}$

$=300$ drums

(b) (300 drums)(\$0.02 per drum) = \$6

75. (a) True, because $h'(x)=f(x)$ and therefore $h''(x)=f'(x)$.

(b) True because h and h' are both differentiable by part (a).

(c) True, because $h'(1)=f(1)=0$.

75. Continued

(d) True, becaue $h'(1) = f(1) = 0$ and $h''(1) = f'(1) < 0$.

(e) False, because $h''(1) = f'(1) < 0$.

(f) False, because $h''(1) = f'(1) \neq 0$

(g) True, because $h'(x) = f(x)$, and f is a decreasing function that includes the point (1,0).

76. Since $f(t)$ is odd, $\int_{-x}^{0} f(t)dt = -\int_{0}^{x} f(t)dt$ because the area between the curve and the x-axis from 0 to x is the opposite of the area between the curve and the x-axis from $-x$ to 0, but it is on the opposite side of the x-axis.

$$\int_0^{-x} f(t)\,dt = -\int_{-x}^{0} f(t)\,dt = -\left[-\int_0^x f(t)dt\right] = \int_0^x f(t)\,dt$$

Thus $\int_0^x f(t)\,dt$ is even.

77. Since $f(t)$ is even, $\int_{-x}^{0} f(t)\,dt = \int_0^x f(t)\,dt$ because the area between the curve and the x-axis from 0 to x is the same as the area between the curve and the x-axis from $-x$ to 0.

$$\int_0^{-x} f(t)\,dt = -\int_{-x}^{0} f(t)\,dt = -\int_0^x f(t)\,dt$$

Thus $\int_0^x f(t)\,dt$ is odd.

78. If f is an even continuous function, then $\int_0^x f(t)\,dt$ is odd, but $\frac{d}{dx}\int_0^x f(t)\,dt = f(x)$. Therefore, f is the derivative of the odd continuous function $\int_0^x f(t)\,dt$.
Similarly, if f is an odd continuous function, then f is the derivative of the even continuous function $\int_0^x f(t)\,dt$.

79. Solving $\text{NINT}\left(\frac{\sin t}{t}, t, 0, x\right) = 1$ graphically, the solution is $x \approx 1.0648397$. We now argue that there are no other solutions, using the functions Si(x) and $f(t)$ as defined in Exercise 56. Since $\frac{d}{dx}\text{Si}(x) = f(x) = \frac{\sin x}{x}$, Si($x$) is increasing on each interval $[2k\pi, (2k+1)\pi]$ and decreasing on each interval $[(2k+1)\pi, (2k+2)\pi]$, where K is a nonnegative integer. Thus, for $x > 0$, Si(x) has its local minima at $x = 2k\pi$, where k is a positive integer. Furthermore, each arch of $y = f(x)$ is smaller in height than the previous one, so $\int_{2k\pi}^{(2k+1)\pi} |f(x)|dx > \int_{(2k+1)\pi}^{(2k+2)\pi} |f(x)|dx$. This means that $\text{Si}(2k+2)\pi) - \text{Si}(2k\pi) = \int_{2k\pi}^{(2k+2)\pi} f(x)\,dx > 0$, so each successive minimum value is greater than the previous one. Since $f(2\pi) \approx \text{NINT}\left(\frac{\sin x}{x}, x, o, 2\pi\right) \approx 1.42$ and Si(x) is continuous for $x > 0$, this means Si$(x) > 1.42$ (and hence Si(x) $\neq$ 1) for $x \geq 2\pi$. Now, Si(x) = 1 has exactly one solution in the interval $[0, \pi]$ because Si(x) is increasing on this interval and $x \approx 1.065$ is a solution. Furthermore, Si(x) = 1 has no solution on the interval $[\pi, 2\pi]$ because Si(x) is decreasing on this interval and Si$(2\pi) \approx 1.42 > 1$. Thus, Si(x) = 1 has exactly one solution in the interval $[0, \infty)$. Also, there is no solution in the interval $(-\infty, 0]$ because Si(x) is odd by Exercise 56 (or 62), which means that Si$(x) \leq 0$ for $x \leq 0$ (since Si$(x) \geq 0$ for $x \geq 0$).

Section 5.5 Trapezoidal Rule (pp. 306–315)

Exploration 1 Area Under a Parabolic Arc

1. Let $y = f(x) = Ax^2 + Bx + C$

Then $y_0 = f(-h) = Ah^2 - Bh + C$,

$y_1 = f(0) = A(0)^2 + B(0) + C = C$, and

$y_2 = f(h) = Ah^2 + Bh + C$.

2.
$$\begin{aligned} y_0 + 4y_1 + y_2 &= Ah^2 - Bh + C + 4C + Ah^2 + Bh + C \\ &= 2Ah^2 + 6C. \end{aligned}$$

3.
$$\begin{aligned} A_p &= \int_{-h}^{h} (Ax^2 + Bx + C)dx \\ &= \left[A\frac{x^3}{3} + B\frac{x^2}{2} + Cx\right]_{-h}^{h} \\ &= A\frac{h^3}{3} + B\frac{h^2}{2} + Ch - \left(-A\frac{h^3}{3} + B\frac{h^2}{2} - Ch\right) \\ &= 2A\frac{h^3}{3} + 2Ch \\ &= \frac{h}{3}(2Ah^2 + 6C) \end{aligned}$$

4. Substitute the expression in step 2 for the parenthetically enclosed expression in step 3:

$$\begin{aligned} A_p &= \frac{h}{3}(2Ah^2 + 6C) \\ &= \frac{h}{3}(y_0 + 4y_1 + y_2). \end{aligned}$$

Quick Review 5.5

1. $y' = -\sin x$
$y'' = -\cos x$
$y'' < 0$ on $[-1, 1]$, so the curve is concave down on $[-1, 1]$.

2. $y' = 4x^3 - 12$
$y'' = 12x^2$
$y'' > 0$ on $[8, 17]$, so the curve is concave up on $[8, 17]$.

3. $y' = 12x^2 - 6x$
$y'' = 24x - 6$
$y'' < 0$ on $[-8, 0]$, so the curve is concave down on $[-8, 0]$.