8. Using $A(0, -4)$, we create the parametric equations $x = 0 + at$ and $y = -4 + bt$, which determine a line passing through A at $t = 0$. We now determine a and b so that the line passes through $B(5, 0)$ at $t = 1$. Since $5 = 0 + a$, we have $a = 5$, and since $0 = -4 + b$, we have $b = 4$. Thus, one parametrization for the line segment is $x = 5t$, $y = -4 + 4t$, $0 \le t \le 1$. (Other answers are possible.)

9. One possible answer: $\frac{\pi}{2} \le t \le \frac{3\pi}{2}$

10. One possible answer: $\frac{3\pi}{2} \le t \le 2\pi$

Section 4.6 Exercises

1. Since $\frac{dA}{dt} = \frac{dA}{dr}\frac{dr}{dt}$, we have $\frac{dA}{dt} = 2\pi r\frac{dr}{dt}$,

2. Since $\frac{dS}{dt} = \frac{dS}{dr}\frac{dr}{dt}$, we have $\frac{dS}{dt} = 8\pi r\frac{dr}{dt}$.

3. (a) Since $\frac{dV}{dt} = \frac{dV}{dh}\frac{dh}{dt}$, we have $\frac{dV}{dt} = \pi r^2\frac{dh}{dt}$.

(b) Since $\frac{dV}{dt} = \frac{dV}{dr}\frac{dr}{dt}$, we have $\frac{dV}{dt} = 2\pi rh\frac{dr}{dt}$.

(c)
$$\frac{dV}{dt} = \frac{d}{dt}\pi r^2 h = \pi\frac{d}{dt}(r^2 h)$$
$$\frac{dV}{dt} = \pi\left(r^2\frac{dh}{dt} + h(2r)\frac{dr}{dt}\right)$$
$$\frac{dV}{dt} = \pi r^2\frac{dh}{dt} + 2\pi rh\frac{dr}{dt}$$

4. (a)
$$\frac{dP}{dt} = \frac{d}{dt}(RI^2)$$
$$\frac{dP}{dt} = R\frac{d}{dt}I^2 + I^2\frac{dR}{dt}$$
$$\frac{dP}{dt} = R\left(2I\frac{dI}{dt}\right) + I^2\frac{dR}{dt}$$
$$\frac{dP}{dt} = 2RI\frac{dI}{dt} + I^2\frac{dR}{dt}$$

(b) If P is constant, we have $\frac{dP}{dt} = 0$, which means

$$2RI\frac{dI}{dt} + I^2\frac{dR}{dt} = 0, \text{ or } \frac{dR}{dt} = -\frac{2R}{I}\frac{dI}{dt} = -\frac{2P}{I^3}\frac{dI}{dt}.$$

5.
$$\frac{ds}{dt} = \frac{d}{dt}\sqrt{x^2 + y^2 + z^2}$$
$$\frac{ds}{dt} = \frac{1}{2\sqrt{x^2 + y^2 + z^2}}\frac{d}{dt}(x^2 + y^2 + z^2)$$
$$\frac{ds}{dt} = \frac{1}{2\sqrt{x^2 + y^2 + z^2}}\left(2x\frac{dx}{dt} + 2y\frac{dy}{dt} + 2z\frac{dz}{dt}\right)$$
$$\frac{ds}{dt} = \frac{x\frac{dx}{dt} + y\frac{dy}{dt} + z\frac{dz}{dt}}{\sqrt{x^2 + y^2 + z^2}}.$$

6.
$$\frac{dA}{dt} = \frac{d}{dt}\left(\frac{1}{2}ab\sin\theta\right)$$
$$\frac{dA}{dt} = \frac{1}{2}\left(\frac{da}{dt} \cdot b \cdot \sin\theta + a \cdot \frac{db}{dt} \cdot \sin\theta + ab \cdot \frac{d}{dt}\sin\theta\right)$$
$$\frac{dA}{dt} = \frac{1}{2}\left(b\sin\theta\frac{da}{dt} + a\sin\theta\frac{db}{dt} + ab\cos\theta\frac{d\theta}{dt}\right)$$

7. (a) Since V is increasing at the rate of 1 volt/sec, $\frac{dV}{dt} = 1$ volt/sec.

(b) Since I is decreasing at the rate of $\frac{1}{3}$ amp/sec, $\frac{dI}{dt} = -\frac{1}{3}$ amp/sec.

(c) Differentiating both sides of $V = IR$, we have

$$\frac{dV}{dt} = I\frac{dR}{dt} + R\frac{dI}{dt}.$$

(d) Note that $V = IR$ gives $12 = 2R$, so $R = 6$ ohms. Now substitute the known values into the equation in (c).

$$1 = 2\frac{dR}{dt} + 6\left(-\frac{1}{3}\right)$$
$$3 = 2\frac{dR}{dt}$$
$$\frac{dR}{dt} = \frac{3}{2}\text{ ohms/sec}$$

R is changing at the rate of $\frac{3}{2}$ ohms/sec. Since this value is positive, R is increasing.

8. Step 1:
r = radius of plate
A = area of plate

Step 2:

At the instant in question, $\frac{dr}{dt} = 0.01$ cm/sec, $r = 50$ cm.

Step 3:

We want to find $\frac{dA}{dt}$.

Step 4:
$A = \pi r^2$

Step 5:
$$\frac{dA}{dt} = 2\pi r\frac{dr}{dt}$$
Step 6:
$$\frac{dA}{dt} = 2\pi(50)(0.01) = \pi\text{ cm}^2/\text{sec}$$
At the instant in question, the area is increasing at the rate of $\pi\text{cm}^2/\text{sec}$.

9. Step 1:
l = length of rectangle
w = width of rectangle
A = area of rectangle
P = perimeter of rectangle
D = length of a diagonal of the rectangle

Step 2:
At the instant in question,
$\frac{dl}{dt} = -2$ cm/sec, $\frac{dw}{dt} = 2$ cm/sec, $l = 12$ cm, and $w = 5$ cm.

Step 3:
We want to find $\frac{dA}{dt}, \frac{dP}{dt}$, and $\frac{dD}{dt}$.

Steps 4, 5, and 6:

(a) $A = lw$

$$\frac{dA}{dt} = l\frac{dw}{dt} + w\frac{dl}{dt}$$
$$\frac{dA}{dt} = (12)(2) + (5)(-2) = 14 \text{ cm}^2/\text{sec}$$

The rate of change of the area is $14 \text{ cm}^2/\text{sec}$.

(b) $P = 2l + 2w$

$$\frac{dP}{dt} = 2\frac{dl}{dt} + 2\frac{dw}{dt}$$
$$\frac{dP}{dt} = 2(-2) + 2(2) = 0 \text{ cm/sec}$$

The rate of change of the perimeter is 0 cm/sec.

(c) $D = \sqrt{l^2 + w^2}$

$$\frac{dD}{dt} = \frac{1}{2\sqrt{l^2+w^2}}\left(2l\frac{dl}{dt} + 2w\frac{dw}{dt}\right) = \frac{l\frac{dl}{dt} + w\frac{dw}{dt}}{\sqrt{l^2+w^2}}$$
$$\frac{dD}{dt} = \frac{(12)(-2)+(5)(2)}{\sqrt{12^2+5^2}} = -\frac{14}{13} \text{ cm/sec}$$

The rate of change of the length of the diameter is $-\frac{14}{13}$ cm/sec.

(d) The area is increasing, because its derivative is positive. The perimeter is not changing, because its derivative is zero. The diagonal length is decreasing, because its derivative is negative.

10. Step 1:
x, y, z = edge lengths of the box
V = volume of the box
S = surface area of the box
s = diagonal length of the box

Step 2:
At the instant in question,
$\frac{dx}{dt} = 1$ m/sec, $\frac{dy}{dt} = -2$ m/sec, $\frac{dz}{dt} = 1$ m/sec, $x = 4$m, $y = 3$m, and $z = 2$m.

Step 3:
We want to find $\frac{dV}{dt}, \frac{dS}{dt}$, and $\frac{ds}{dt}$.

Steps 4, 5, and 6:

(a) $V = xyz$

$$\frac{dV}{dt} = xy\frac{dz}{dt} + xz\frac{dy}{dt} + yz\frac{dx}{dt}$$
$$\frac{dV}{dt} = (4)(3)(1) + (4)(2)(-2) + (3)(2)(1) = 2 \text{ m}^3/\text{sec}$$

The rate of change of the volume is $2 \text{ m}^3/\text{sec}$.

(b) $S = 2(xy + xz + yz)$

$$\frac{dS}{dt} 2\left(x\frac{dy}{dt} + y\frac{dx}{dt} + x\frac{dz}{dt} + z\frac{dx}{dt} + y\frac{dz}{dt} + z\frac{dy}{dt}\right)$$
$$\frac{dS}{dt} = 2[(4)(-2) + (3)(1) + (4)(1)$$
$$+ (2)(1) + (3)(1) + (2)(-2)] = 0 \text{ m}^2/\text{sec}$$

The rate of change of the surface area is $0 \text{ m}^2/\text{sec}$.

(c) $s = \sqrt{x^2 + y^2 + z^2}$

$$\frac{ds}{dt} = \frac{1}{2\sqrt{x^2+y^2+z^2}}\left(2x\frac{dx}{dt} + 2y\frac{dy}{dt} + 2z\frac{dz}{dy}\right)$$
$$= \frac{x\frac{dx}{dt} + y\frac{dy}{dt} + z\frac{dz}{dt}}{\sqrt{x^2+y^2+z^2}}$$
$$\frac{ds}{dt} = \frac{(4)(1)+(3)(-2)+(2)(1)}{\sqrt{4^2+3^2+2^2}} = \frac{0}{\sqrt{29}} = 0 \text{ m/sec}$$

The rate of change of the diagonal length is 0 m/sec.

11. Step 1:
r = radius of spherical balloon
S = surface area of spherical balloon
V = volume of spherical balloon

Step 2:
At the instant in question, $\frac{dV}{dt} = 100\pi \text{ ft}^3/\text{min}$ and $r = 5$ ft.

Step 3:
We want to find the values of $\frac{dr}{dt}$ and $\frac{dS}{dt}$.

Steps 4, 5, and 6:

(a) $V = \frac{4}{3}\pi r^3$

$$\frac{dV}{dt} = 4\pi r^2\frac{dr}{dt}$$
$$100\pi = 4\pi(5)^2\frac{dr}{dt}$$
$$\frac{dr}{dt} = 1 \text{ ft/min}$$

The radius is increasing at the rate of 1 ft/min.

11. Continued

(b) $S = 4\pi r^2$

$$\frac{dS}{dt} = 8\pi r \frac{dr}{dt}$$

$$\frac{dS}{dt} = 8\pi(5)(1)$$

$$\frac{dS}{dt} = 40\pi \text{ ft}^2/\text{min}$$

The surface area is increasing at the rate of 40π ft^2/min.

12. Step 1:

r = radius of spherical droplet
S = surface area of spherical droplet
V = volume of spherical droplet

Step 2:

No numerical information is given.

Step 3:

We want to show that $\frac{dr}{dt}$ is constant.

Step 4:

$S = 4\pi r^2$, $V = \frac{4}{3}\pi r^3$, $\frac{dV}{dt} = kS$ for some constant k

Steps 5 and 6:

Differentiating $V = \frac{4}{3}\pi r^3$, we have $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$.

Substituting kS for $\frac{dV}{dt}$ and S for $4\pi r^2$, we have $kS = S\frac{dr}{dt}$, or $\frac{dr}{dt} = k$.

13. Step 1:

s = (diagonal) distance from antenna to airplane
x = horizontal distance from antenna to airplane

Step 2:

At the instant in question,

$s = 10$ mi and $\frac{ds}{dt} = 300$ mph.

Step 3:

We want to find $\frac{dx}{dt}$.

Step 4:

$x^2 + 49 = s^2$ or $x = \sqrt{s^2 - 49}$

Step 5:

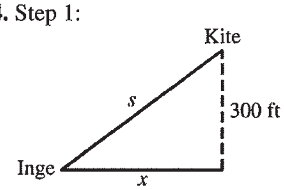
$$\frac{dx}{dt} = \frac{1}{2\sqrt{s^2-49}}\left(2s\frac{ds}{dt}\right) = \frac{s}{\sqrt{s^2-49}}\frac{ds}{dt}$$

Step 6:

$$\frac{dx}{dt} = \frac{10}{\sqrt{10^2-49}}(300) = \frac{3000}{\sqrt{51}} \text{ mph} \approx 420.08 \text{ mph}$$

The speed of the airplane is about 420.08 mph.

14. Step 1:



s = length of kite string
x = horizontal distance from Inge to kite

Step 2:

At the instant in question, $\frac{dx}{dt} = 25$ ft/sec and $s = 500$ ft

Step 3:

We want to find $\frac{ds}{dt}$.

Step 4:

$x^2 + 300^2 = s^2$

Step 5:

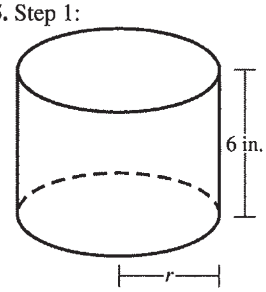
$2x\frac{dx}{dt} = 2s\frac{ds}{dt}$ or $x\frac{dx}{dt} = s\frac{ds}{dt}$

Step 6:

At the instant in question, since $x^2 + 300^2 = s^2$, we have $x = \sqrt{s^2 - 300^2} = \sqrt{500^2 - 300^2} = 400$.

Thus $(400)(25) = (500)\frac{ds}{dt}$, so $\frac{ds}{dt}$, so $\frac{ds}{dt} = 20$ ft/sec. Inge must let the string out at the rate of 20 ft/sec.

15. Step 1:



The cylinder shown represents the shape of the hole.

r = radius of cylinder
V = volume of cylinder

Step 2:

At the instant in question, $\frac{dr}{dt} = \frac{0.001 \text{ in.}}{3 \text{ min}} = \frac{1}{3000}$ in./min and (since the diameter is 3.800 in.), $r = 1.900$ in.

Step 3:

We want to find $\frac{dV}{dt}$.

Step 4:

$V = \pi r^2(6) = 6\pi r^2$

Step 5:

$$\frac{dV}{dt} = 12\pi r \frac{dr}{dt}$$

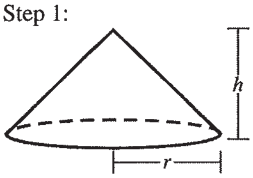
15. Continued

Step 6:

$$\frac{dV}{dt}=12\pi(1.900)\left(\frac{1}{3000}\right)=\frac{19\pi}{2500}=0.0076\pi$$
$$\approx 0.0239\ \text{in}^3/\text{min}.$$

The volume is increasing at the rate of approximately $0.0239\ \text{in}^3/\text{min}$.

16. Step 1:



r = base radius of cone
h = height of cone
V = volume of cone

Step 2:

At the instant in question, $h = 4$ m and $\frac{dV}{dt}=10\ \text{m}^3/\text{min}$.

Step 3:

We want to find $\frac{dh}{dt}$ and $\frac{dr}{dt}$.

Step 4:

Since the height is $\frac{3}{8}$ of the base diameter, we have

$h=\frac{3}{8}(2r)$ or $r=\frac{4}{3}h$.

We also have $V=\frac{1}{3}\pi r^2 h=\frac{1}{3}\pi\left(\frac{4}{3}h\right)^2 h=\frac{16\pi h^3}{27}$. We will use the equations $V=\frac{16\pi h^3}{27}$ and $r=\frac{4}{3}h$.

Step 5 and 6:

(a) $\frac{dV}{dt}=\frac{16\pi h^2}{9}\frac{dh}{dt}$

$$10=\frac{16\pi(4)^2}{9}\frac{dh}{dt}$$
$$\frac{dh}{dt}=\frac{45}{128\pi}\ \text{m/min}=\frac{1125}{32\pi}\ \text{cm/min}$$

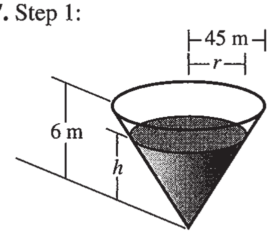
The height is changing at the rate of $\frac{1125}{32\pi}\approx 11.19$ cm / min.

(b) Using the results from Step 4 and part (a), we have

$$\frac{dr}{dt}=\frac{4}{3}\frac{dh}{dt}=\frac{4}{3}\left(\frac{1125}{32\pi}\right)=\frac{375}{8\pi}\ \text{cm/min}.$$

The radius is changing at the rate of $\frac{375}{8\pi}\approx 14.92$ cm/min.

17. Step 1:



r = radius of top surface of water
h = depth of water in reservoir
V = volume of water in reservoir

Step 2:

At the instant in question, $\frac{dV}{dt}=-50\ \text{m}^3/\text{min}$ and $h = 5$ m.

Step 3:

We want to find $-\frac{dh}{dt}$ and $\frac{dr}{dt}$.

Step 4:

Note that $\frac{h}{r}=\frac{6}{45}$ by similar cones, so $r = 7.5h$.

Then $V=\frac{1}{3}\pi r^2 h=\frac{1}{3}\pi(7.5h)^2 h=18.75\pi h^3$

Steps 5 and 6:

(a) Since $V=18.75\pi h^3$, $\frac{dV}{dt}=56.25\pi h^2\frac{dh}{dt}$.

Thus $-50=56.25\pi(5^2)\frac{dh}{dt}$, and

so $\frac{dh}{dt}=-\frac{8}{225\pi}$ m/min $=-\frac{32}{9\pi}$ cm/min.

The water level is falling by $\frac{32}{9\pi}\approx 1.13$ cm/min.

(Since $\frac{dh}{dt}<0$, the rate at which the water level is *falling* is positive.)

(b) Since $r = 7.5h$, $\frac{dr}{dt}=7.5\frac{dh}{dt}=-\frac{80}{3\pi}$ cm/min. The rate of change of the radius of the water's surface is $-\frac{80}{3\pi}\approx -8.49$ cm/min.

18. (a) Step 1:

y = depth of water in bowl
V = volume of water in bowl

Step 2:

At the instant in question, $\frac{dV}{dt}=-6\ \text{m}^3/\text{min}$ and $y = 8$ m.

Step 3:

We want to find the value of $\frac{dy}{dt}$.

Step 4:

$V=\frac{\pi}{3}y^2(39-y)$ or $V=13\pi y^2-\frac{\pi}{3}y^3$

18. Continued

Step 5:

$$\frac{dV}{dt}=(26\pi y-\pi y^2)\frac{dy}{dt}$$

Step 6:

$$-6=\left[26\pi(8)-\pi(8^2)\right]\frac{dy}{dt}$$

$$-6=144\pi\frac{dy}{dt}$$

$$\frac{dy}{dt}=-\frac{1}{24\pi}\approx-0.01326\text{ m/min}$$

$$\text{or }-\frac{25}{6\pi}\approx-1.326\text{ cm/min}$$

(b) Since $r^2+(13-y)^2=13^2$,

$$r=\sqrt{169-(13-y)^2}=\sqrt{26y-y^2}.$$

(c) Step 1:

y = depth of water

r = radius of water surface

V = volume of water in bowl

Step 2:

At the instant in question, $\frac{dV}{dt}=-6\ \text{m}^3/\text{min}$, $y=8$ m,

and therefore (from part (a)) $\frac{dy}{dt}=-\frac{1}{24\pi}$ m/min.

Step 3:

We want to find the value of $\frac{dr}{dt}$.

Step 4:

From part (b), $r=\sqrt{26y-y^2}$.

Step 5:

$$\frac{dr}{dt}=\frac{1}{2\sqrt{26y-y^2}}(26-2y)\frac{dy}{dt}=\frac{13-y}{\sqrt{26y-y^2}}\frac{dy}{dt}$$

Step 6:

$$\frac{dr}{dt}=\frac{13-8}{\sqrt{26(8)-8^2}}\left(\frac{1}{-24\pi}\right)=\frac{5}{12}\left(-\frac{1}{24\pi}\right)$$

$$=-\frac{5}{288\pi}\approx-0.00553\text{ m/min}$$

$$\text{or }-\frac{125}{72\pi}\approx-0.553\text{ cm/min}$$

19. Step 1:

x = distance from wall to base of ladder

y = height of top of ladder

A = area of triangle formed by the ladder, wall, and ground

θ = angle between the ladder and the ground

Step 2:

At the instant in question, $x=12$ ft and $\frac{dx}{dt}=5$ ft/sec.

Step 3:

We want to find $-\frac{dy}{dt}$, $\frac{dA}{dt}$, and $\frac{d\theta}{dt}$.

Steps 4, 5, and 6:

(a) $x^2+y^2=169$

$$2x\frac{dx}{dt}+2y\frac{dy}{dt}=0$$

To evaluate, note that, at the instant in question,

$$y=\sqrt{169-x^2}=\sqrt{169-12^2}=5.$$

Then $2(12)(5)+2(5)\frac{dy}{dt}=0$

$$\frac{dy}{dt}=-12\text{ ft/sec}\left(\text{or }-\frac{dy}{dt}=12\text{ ft/sec}\right)$$

The top of the ladder is sliding down the wall at the rate of 12 ft/sec. (Note that the *downward* rate of motion is positive.)

(b) $A=\frac{1}{2}xy$

$$\frac{dA}{dt}=\frac{1}{2}\left(x\frac{dy}{dt}+y\frac{dx}{dt}\right)$$

Using the results from step 2 and from part (a), we have $\frac{dA}{dt}=\frac{1}{2}[(12)(-12)+(5)(5)]=-\frac{119}{2}\ \text{ft}^2/\text{sec}$. The area of the triangle is changing at the rate of $-59.5\ \text{ft}^2/\text{sec}$.

(c) $\tan\theta=\frac{y}{x}$

$$\sec^2\theta\frac{d\theta}{dt}=\frac{x\frac{dy}{dt}-y\frac{dx}{dt}}{x^2}$$

Since $\tan\theta=\frac{5}{12}$, we have

$\left(\text{for }0\le\theta<\frac{\pi}{2}\right)\cos\theta=\frac{12}{13}$ and so $\sec^2\theta\frac{1}{\left(\frac{12}{13}\right)^2}=\frac{169}{144}$.

Combining this result with the results from step 2 and from part (a), we have $\frac{169}{144}\frac{d\theta}{dt}=\frac{(12)(-12)-(5)(5)}{12^2}$, so

$\frac{d\theta}{dt}=-1$ radian/sec. The angle is changing at the rate of -1 radian/sec.

20. Step 1:

h = height (or depth) of the water in the trough

V = volume of water in the trough

Step 2:

At the instant in question, $\frac{dV}{dt}=2.5\ \text{ft}^3/\text{min}$ and $h=2$ ft.

20. Continued

Step 3:

We want to find $\frac{dh}{dt}$.

Step 4:

The width of the top surface of the water is $\frac{4}{3}h$, so we have $V = \frac{1}{2}(h)\left(\frac{4}{3}h\right)(15)$, or $V = 10h^2$

Step 5:

$$\frac{dV}{dt} = 20h\frac{dh}{dt}$$

Step 6:

$$2.5 = 20(2)\frac{dh}{dt}$$

$$\frac{dh}{dt} = 0.0625 = \frac{1}{16} \text{ ft/min}$$

The water level is increasing at the rate of $\frac{1}{16}$ ft/min.

21. Step 1:

l = length of rope

x = horizontal distance from boat to dock

θ = angle between the rope and a vertical line

Step 2:

At the instant in question, $\frac{dl}{dt} = -2$ ft/sec and $l = 10$ ft.

Step 3:

We want to find the values of $-\frac{dx}{dt}$ and $\frac{d\theta}{dt}$.

Steps 4, 5, and 6:

(a) $x = \sqrt{l^2 - 36}$

$$\frac{dx}{dt} = \frac{l}{\sqrt{l^2 - 36}}\frac{dl}{dt}$$

$$\frac{dx}{dt} = \frac{10}{\sqrt{10^2 - 36}}(-2) = -2.5 \text{ ft/sec}$$

The boat is approaching the dock at the rate of 2.5 ft/sec.

(b) $\theta = \cos^{-1}\frac{6}{l}$

$$\frac{d\theta}{dt} = -\frac{1}{\sqrt{1-\left(\frac{6}{l}\right)^2}}\left(-\frac{6}{l^2}\right)\frac{dl}{dt}$$

$$\frac{d\theta}{dt} = -\frac{1}{\sqrt{1-0.6^2}}\left(-\frac{6}{10^2}\right)(-2) = -\frac{3}{20} \text{ radian/sec}$$

The rate of change of angle θ is $-\frac{3}{20}$ radian/sec.

22. Step 1:

x = distance from origin to bicycle

y = height of balloon (distance from origin to balloon)

s = distance from balloon to bicycle

Step 2:

We know that $\frac{dy}{dt}$ is a constant 1 ft/sec and $\frac{dx}{dt}$ is a constant 17 ft/sec. Three seconds before the instant in question, the values of x and y are $x = 0$ ft and $y = 65$ ft. Therefore, at the instant in question $x = 51$ ft and $y = 68$ ft.

Step 3:

We want to find the value of $\frac{ds}{dt}$ at the instant in question.

Step 4:

$$s = \sqrt{x^2 + y^2}$$

Step 5:

$$\frac{ds}{dt} = \frac{1}{2\sqrt{x^2+y^2}}\left(2x\frac{dx}{dt} + 2y\frac{dy}{dt}\right) = \frac{x\frac{dx}{dt} + y\frac{dy}{dt}}{\sqrt{x^2+y^2}}$$

Step 6:

$$\frac{ds}{dt} = \frac{(51)(17)+(68)(1)}{\sqrt{51^2+68^2}} = 11 \text{ ft/sec}$$

The distance between the balloon and the bicycle is increasing at the rate of 11 ft/sec.

23. $\frac{dy}{dt} = \frac{dy}{dt}\frac{dx}{dt} = -10(1+x^2)^{-2}(2x)\frac{dx}{dt} = -\frac{20x}{(1+x^2)^2}\frac{dx}{dt}$

Since $\frac{dx}{dt} = 3$cm/sec, we have

$$\frac{dy}{dt} = -\frac{60x}{(1+x^2)^2} \text{ cm/sec.}$$

(a) $\frac{dy}{dt} = -\frac{60(-2)}{[1+(-2)^2]^2} = \frac{120}{5^2} = \frac{24}{5}$ cm/sec

(b) $\frac{dy}{dt} = -\frac{60(0)}{(1+0^2)^2} = 0$ cm/sec

(c) $\frac{dy}{dt} = -\frac{60(20)}{(1+20^2)^2} \approx -0.00746$ cm/sec

24. $\frac{dy}{dt} = \frac{dy}{dx}\frac{dx}{dt} = (3x^2 - 4)\frac{dx}{dt}$

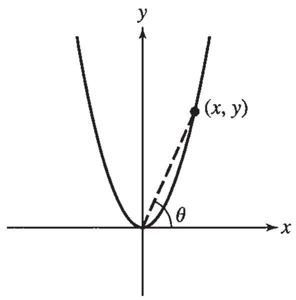
Since $\frac{dx}{dt} = -2$ cm/sec, we have $\frac{dy}{dt} = 8 - 6x^2$ cm/sec.

(a) $\frac{dy}{dt} = 8 - 6(-3)^2 = -46$ cm/sec

(b) $\frac{dy}{dt} = 8 - 6(1)^2 = 2$ cm/sec

(c) $\frac{dy}{dt} = 8 - 6(4)^2 = -88$ cm/sec

25. Step 1:



$x = x$-coordinate of particle's location
$y = y$-coordinate of particle's location
θ = angle of inclination of line joining the particle to the origin.

Step 2:
At the instant in question,
$\frac{dx}{dt} = 10$ m/sec and $x = 3$ m.

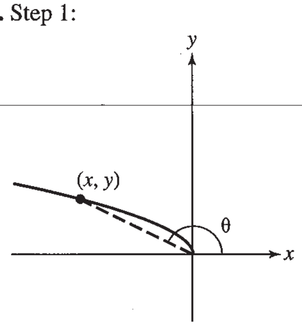
Step 3:
We want to find $\frac{d\theta}{dt}$.

Step 4:
Since $y = x^2$, we have $\tan\,\theta = \frac{y}{x} = \frac{x^2}{x} = x$ and so, for $x > 0$,
$\theta = \tan^{-1} x$.

Step 5:
$$\frac{d\theta}{dt} = \frac{1}{1+x^2}\frac{dx}{dt}$$

Step 6:
$$\frac{d\theta}{dt} = \frac{1}{1+3^2}(10) = 1 \text{ radian/sec}$$
The angle of inclination is increasing at the rate of 1 radian/sec.

26. Step 1:



$x = x$-coordinate of particle's location
$y = y$-coordinate of particle's location
θ = angle of inclination of line joining the particle to the origin

Step 2:
At the instant in question, $\frac{dx}{dt} = -8$ m / sec and $x = -4$ m.

Step 3:
We want to find $\frac{d\theta}{dt}$,

Step 4:
Since $y = \sqrt{-x}$, we have $\tan\theta = \frac{y}{x} = \frac{\sqrt{-x}}{x} - (-x)^{-1/2}$,
and so, for $x < 0$,
$\theta = \pi + \tan^{-1}[-(-x)^{1/2}] = \pi - \tan^{-1}(-x)^{-1/2}$.

Step 5:
$$\frac{d\theta}{dt} = -\frac{1}{1+[(-x)^{-1/2}]^2}\left((-\frac{1}{2}(-x)^{-3/2}(-1)\right)\frac{dx}{dt}$$
$$= -\frac{1}{1-\left(\frac{1}{x}\right)}\frac{1}{2(-x)^{3/2}}\frac{dx}{dt}$$
$$= \frac{1}{2\sqrt{-x}(x-1)}\frac{dx}{dt}$$

Step 6:
$$\frac{d\theta}{dt} = \frac{1}{2\sqrt{4}(-4-1)}(-8) = \frac{2}{5} \text{ radian/sec}$$
The angle of inclination is increasing at the rate of $\frac{2}{5}$ radian/sec.

27. Step 1:
r = radius of balls plus ice
S = surface area of ball plus ice
V = volume of ball plus ice

Step 2:
At the instant in question,
$\frac{dV}{dt} = -8 \text{ mL/min} = -8 \text{ cm}^3/\text{min and } r = \frac{1}{2}(20) = 10 \text{ cm}.$

Step 3:
We want to find $-\frac{dS}{dt}$.

Step 4:
We have $V = \frac{4}{3}\pi r^3$ and $S = 4\pi r^2$. These equations can be combined by noting that $r = \left(\frac{3V}{4\pi}\right)^{1/3}$, so $S = 4\pi\left(\frac{3V}{4\pi}\right)^{2/3}$

Step 5:
$$\frac{dS}{dt} = 4\pi\left(\frac{2}{3}\right)\left(\frac{3V}{4\pi}\right)^{-1/3}\left(\frac{3}{4\pi}\right)\frac{dV}{dt} = 2\left(\frac{3V}{4\pi}\right)^{-1/3}\frac{dV}{dt}$$

Step 6:
Note that $V = \frac{4}{3}\pi(10)^3 = \frac{4000\pi}{3}$.
$$\frac{dS}{dt} = 2\left(\frac{3}{4\pi}\cdot\frac{4000\pi}{3}\right)^{-1/3}(-8) = \frac{-16}{\sqrt[3]{1000}} = -1.6 \text{ cm}^2/\text{min}$$
Since $\frac{dS}{dt} < 0$, the rate of *decrease* is positive. The surface area is decreasing at the rate of $1.6 \text{ cm}^2/\text{min}$.

28. Step 1:

x = x-coordinate of particle

y = y-coordinate of particle

D = distance from origin to particle

Step 2:

At the instant in question, $x = 5$ m, $y = 12$ m, $\frac{dx}{dt} = -1$ m/sec, and $\frac{dy}{dt} = -5$ m/sec.

Step 3:

We want to find $\frac{dD}{dt}$.

Step 4:

$D = \sqrt{x^2 + y^2}$

Step 5:

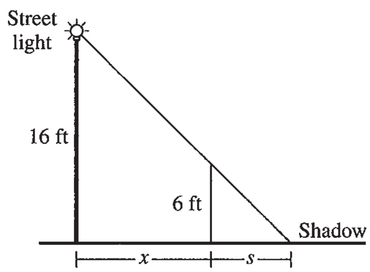
$$\frac{dD}{dt} = \frac{1}{2\sqrt{x^2+y^2}}\left(2x\frac{dx}{dt} + 2y\frac{dy}{dt}\right) = \frac{x\frac{dx}{dt} + y\frac{dy}{dt}}{\sqrt{x^2+y^2}}$$

Step 6:

$$\frac{dD}{dt} = \frac{(5)(-1)+(12)(-5)}{\sqrt{5^2+12^2}} = -5 \text{ m/sec}$$

The particle's distance from the origin is changing at the rate of –5 m/sec.

29. Step 1:



x = distance from streetlight base to man

s = length of shadow

Step 2:

At the instant in question, $\frac{dx}{dt} = -5$ ft/sec and $x = 10$ ft.

Step 3:

We want to find $\frac{ds}{dt}$.

Step 4:

By similar triangles, $\frac{s}{6} = \frac{s+x}{16}$. This is equivalent to $16s = 6s + 6x$, or $s = \frac{3}{5}x$.

Step 5:

$\frac{ds}{dt} = \frac{3}{5}\frac{dx}{dt}$

Step 6:

$\frac{ds}{dt} = \frac{3}{5}(-5) = -3$ ft/sec

The shadow length is changing at the rate of –3 ft/sec.

30. Step 1:

s = distance ball has fallen

x = distance from bottom of pole to shadow

Step 2:

At the instant in question, $s = 16\left(\frac{1}{2}\right)^2 = 4$ ft and $\frac{ds}{dt} = 32\left(\frac{1}{2}\right) = 16$ ft/sec.

Step 3:

We want to find $\frac{dx}{dt}$.

Step 4:

By similar triangles, $\frac{x-30}{50-s} = \frac{x}{50}$. This is equivalent to $50x - 1500 = 50x - sx$, or $sx = 1500$. We will use $x = 1500s^{-1}$.

Step 5 :

$\frac{dx}{dt} = -500s^{-2}\frac{ds}{dt}$

Step 6:

$\frac{dx}{dt} = -1500(4)^{-2}(16) = -1500$ ft/sec

The shadow is moving at a velocity of –1500 ft/sec.

31. Step 1:

x = position of car ($x = 0$ when car is right in front of you)

θ = camera angle. (We assume θ is negative until the car passess in front of you, and then positive.)

Step 2:

At the first instant in question, $x = 0$ ft and $\frac{dx}{dt} = 264$ ft/sec.

A half second later, $x = \frac{1}{2}(264) = 132$ ft and $\frac{dx}{dt} = 264$ ft/sec.

Step 3:

We want to find $\frac{d\theta}{dt}$ at each of the two instants.

Step 4:

$\theta = \tan^{-1}\left(\frac{x}{132}\right)$

Step 5:

$$\frac{d\theta}{dt} = \frac{1}{1+\left(\frac{x}{132}\right)^2} \bullet \frac{1}{132}\frac{dx}{dt}$$

31. Continued

Step 6:

When $x = 0: \dfrac{d\theta}{dt} = \dfrac{1}{1+\left(\dfrac{0}{132}\right)^2}\left(\dfrac{1}{132}\right)(264) = 2$ radians/sec

When $x = 132: \dfrac{d\theta}{dt} = \dfrac{1}{1+\left(\dfrac{132}{132}\right)^2}\left(\dfrac{1}{132}\right)(264) = 1$ radians/sec

32. Step 1:

$p = x$-coordinate of plane's position

$x = x$-coordinate of car's position

$s =$ distance from plane to car (line-of-sight)

Step 2:

At the instant in question,

$p = 0, \dfrac{dp}{dt} = 120$ mph, $s = 5$ mi, and $\dfrac{ds}{dt} = -160$ mph.

Step 3:

We want to find $-\dfrac{dx}{dt}$.

Step 4:

$(x-p)^2 + 3^2 = s^2$

Step 5:

$$2(x-p)\left(\frac{dx}{dt} - \frac{dp}{dt}\right) = 2s\frac{ds}{dt}$$

Step 6:

Note that, at the instant in question,

$x = \sqrt{5^2 - 3^2} = 4$ mi.

$$2(4-0)\left(\frac{dx}{dt} - 120\right) = 2(5)(-160)$$

$$8\left(\frac{dx}{dt} - 120\right) = -1600$$

$$\frac{dx}{dt} - 120 = -200$$

$$\frac{dx}{dt} = -80 \text{ mph}$$

The car's speed is 80 mph.

33. Step 1:

$s =$ shadow length

$\theta =$ sun's angle of elevation

Step 2:

At the instant in question,

$s = 60$ ft and $\dfrac{d\theta}{dt} = 0.27°/\text{min} = 0.0015\pi$ radian/min.

Step 3:

We want to find $-\dfrac{ds}{dt}$.

Step 4:

$\tan\theta = \dfrac{80}{s}$ or $s = 80\cot\theta$

Step 5:

$$\frac{ds}{dt} = -80\csc^2\theta\frac{d\theta}{dt}$$

Step 6:

Note that, at the moment in question, since

$\tan\theta = \dfrac{80}{60}$ and $0 < \theta < \dfrac{\pi}{2}$, we have $\sin\theta = \dfrac{4}{5}$ and so

$\csc\theta = \dfrac{5}{4}$.

$$\begin{aligned}\frac{ds}{dt} &= -80\left(\frac{5}{4}\right)^2(0.0015\pi)\\ &= -0.1875\pi\,\frac{\text{ft}}{\text{min}}\bullet\frac{12\text{ in}}{1\text{ ft}}\\ &= -2.25\pi \text{ in./min}\\ &\approx -7.1 \text{ in./min}\end{aligned}$$

Since $\dfrac{ds}{dt} < 0$, the rate at which the shadow length is *decreasing* is positive. The shadow length is decreasing at the rate of approximately 7.1 in./min.

34. Step 1:

$a =$ distance from origin to A

$b =$ distance from origin to B

$\theta =$ angle shown in problem statement

Step 2:

At the instant in question, $\dfrac{da}{dt} = -2\text{m/sec}, \dfrac{db}{dt} = 1$ m/sec,

$a = 10$m, and $b = 20$m.

Step 3:

We want to find $\dfrac{d\theta}{dt}$.

Step 4:

$\tan\theta = \dfrac{a}{b}$ or $\theta = \tan^{-1}\left(\dfrac{a}{b}\right)$

Step 5:

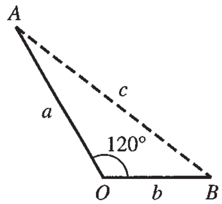
$$\frac{d\theta}{dt} = \frac{1}{1+\left(\frac{a}{b}\right)^2}\,\frac{b\frac{da}{dt} - a\frac{db}{dt}}{b^2} = \frac{b\frac{da}{dt} - a\frac{db}{dt}}{a^2+b^2}$$

Step 6:

$$\begin{aligned}\frac{d\theta}{dt} = \frac{(20)(-2)-(10)(1)}{10^2+20^2} &= -0.1 \text{ radian/sec}\\ &\approx -5.73 \text{ degrees/sec}\end{aligned}$$

To the nearest degree, the angle is changing at the rate of –6 degrees per second.

35. Step 1:



a = distance from O to A
b = distance from O to B
c = distance from A to B

Step 2:
At the instant in question, $a = 5$ nautical miles, $b = 3$ nautical miles, $\frac{da}{dt} = 14$ knots, and $\frac{db}{dt} = 21$ knots.

Step 3:

We want to find $\frac{dc}{dt}$,

Step 4:

Law of Cosines : $c^2 = a^2 + b^2 - 2ab\cos 120°$

$$c^2 = a^2 + b^2 + ab$$

Step 5:

$$2c\frac{dc}{dt} = 2a\frac{da}{dt} + 2b\frac{db}{dt} + a\frac{db}{dt} + b\frac{da}{dt}$$

Step 6:
Note that, at the instant in question,

$$c = \sqrt{a^2 + b^2 + ab} = \sqrt{(5)^2 + (3)^2 + (5)(3)} = \sqrt{49} = 7$$

$$2(7)\frac{dc}{dt} = 2(5)(14) + 2(3)(21) + (5)(21) + (3)(14)$$
$$14\frac{dc}{dt} = 413$$
$$\frac{dc}{dt} = 29.5 \text{ knots}$$

The ships are moving apart at a rate of 29.5 knots.

36. True. Since $\frac{dC}{dt} = 2\pi\frac{dr}{dt}$, a constant $\frac{dr}{dt}$ results in a constant $\frac{dC}{dt}$.

37. False. Since $\frac{dA}{dt} = 2\pi r\frac{dr}{dt}$, the value of $\frac{dA}{dt}$ depends on r.

38. A. $V = s^3$
$$\frac{dV}{dt} = 3s^2\frac{ds}{dt}$$
$$24 = 3s^2(2)$$
$$s = 2 \text{ in}$$

39. E. $sA = 6s^2$
$$\frac{dsA}{dt} = 12s\frac{ds}{dt}$$
$$12 = 12s\frac{ds}{dt}$$
$$\frac{ds}{dt} = \frac{1}{s}$$
$$V = s^3$$
$$\frac{dV}{dt} = 3s^2\frac{ds}{dt} = 3s^2\frac{1}{s}$$
$$24 = 3s$$
$$s = 8 \text{ in}$$

40. C. $x^2 + y^2 = 1$
$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$
$$x\frac{dx}{dt} = -y\frac{dy}{dt}$$
$$\frac{x}{-y}\frac{dx}{dt} = \frac{dy}{dt}$$
$$\left(\frac{0.6}{-0.8}\right)3 = \frac{dy}{dt}$$
$$\frac{dy}{dt} = -2.25.$$

41. B. $v = \pi r^2 l$
$$\frac{dv}{dt} = 2\pi rl\frac{dr}{dt} + \pi r^2\frac{dl}{dt}$$
$$0 = 2\pi(1)(100)\frac{dr}{dt} + \pi(1)^2 2$$
$$\frac{dr}{dt} = \frac{-2\pi}{200\pi}$$
$$\frac{dr}{dt} = -.01 \text{ cm/s}$$

42. **(a)** Note that the level of the coffee in the cone is not needed until part (b).

Step 1:

V_1 = volume of coffee in pot

y = depth of coffee in pot

Step 2:

$$\frac{dV_1}{dt} = 10 \text{ in}^3/\text{min}$$

Step 3:

We want to find the value of $\frac{dy}{dt}$.

Step 4:
$V_1 = 9\pi y$
Step 5:
$$\frac{dV_1}{dt} = 9\pi\frac{dy}{dt}$$

42. Continued

Step 6:

$$10 = 9\pi \frac{dy}{dt}$$

$$\frac{dy}{dt} = \frac{10}{9\pi} \approx 0.354 \text{ in./min}$$

The level in the pot is increasing at the rate of approximately 0.354 in./min.

(b) Step 1:

V_2 = volume of coffee in filter

r = radius of surface of coffee in filter

h = depth of coffee in filter

Step 2:

At the instant in question, $\frac{dV_2}{dt} = -10 \text{ in}^3/\text{min}$ and $h = 5$ in.

Step 3:

We want to find $-\frac{dh}{dt}$.

Step 4:

Note that $\frac{r}{h} = \frac{3}{6}$, so $r = \frac{h}{2}$.

Then $V_2 = \frac{1}{3}\pi r^2 h = \frac{\pi h^3}{12}$.

Step 5:

$$\frac{dV_2}{dt} = \frac{\pi h^2}{4}\frac{dh}{dt}$$

Step 6:

$$-10 = \frac{\pi(5)^2}{4}\frac{dh}{dt}$$

$$\frac{dh}{dt} = -\frac{8}{5\pi} \text{ in./min}$$

Note that $\frac{dh}{dt} < 0$, so the rate at which the level is *falling* is positive. The level in the come is falling at the rate of $\frac{8}{5\pi} \approx 0.509$ in./min.

43. (a) $\frac{dc}{dt} = \frac{d}{dt}(x^3 - 6x^2 + 15x)$

$$= (3x^2 - 12x + 15)\frac{dx}{dt}$$

$$= [3(2)^2 - 12(2) + 15](0.1)$$

$$= 0.3$$

$$\frac{dr}{dt} = \frac{d}{dt}(9x) = 9\frac{dx}{dt} = 9(0.1) = 0.9$$

$$\frac{dp}{dt} = \frac{dr}{dt} - \frac{dc}{dt} = 0.9 - 0.3 = 0.6$$

(b) $\frac{dc}{dt} = \frac{d}{dt}\left(x^3 - 6x^2 + \frac{45}{x}\right)$

$$= \left(3x^2 - 12x - \frac{45}{x^2}\right)\frac{dx}{dt}$$

$$= \left[3(1.5)^2 - 12(1.5) - \frac{45}{1.5^2}\right](0.05)$$

$$= -1.5625$$

$$\frac{dr}{dt} = \frac{d}{dt}(70x) = 70\frac{dx}{dt} = 70(0.05) = 3.5$$

$$\frac{dp}{dt} = \frac{dr}{dt} - \frac{dc}{dt} = 3.5 - (-1.5625) = 5.0625$$

44. Step 1:

Q = rate of CO_2 exhalation (mL/min)

D = difference between CO_2 concentration in blood pumped to the lungs and CO_2 concentration in blood returning from the lungs (mL/L)

y = cardiac output

Step 2:

At the instant in question, $Q = 233$ mL/min, $D = 41$ mL/L, $\frac{dD}{dt} = -2$ (mL/L)/min, and $\frac{dQ}{dt} = 0 \text{ mL/min}^2$.

Step 3:

We want to find the value of $\frac{dy}{dt}$.

Step 4:

$$y = \frac{Q}{D}$$

Step 5:

$$\frac{dy}{dt} = \frac{D\frac{dQ}{dt} - Q\frac{dD}{dt}}{D^2}$$

Step 6:

$$\frac{dy}{dt} = \frac{(41)(0) - (233)(-2)}{(41)^2} = \frac{466}{1681} \approx 0.277 \text{ L/min}^2$$

The cardiac output is increasing at the rate of approximately 0.277 L/min^{-2}.

45. (a) The point being plotted would correspond to a point on the edge of the wheel as the wheel turns.

(b) One possible answer is $\theta = 16\pi t$, where t is in seconds. (An arbitrary constant may be added to this expression, and we have assumed counterclockwise motion.)

45. Continued

(c) In general, assuming counterclockwise motion:

$$\frac{dx}{dt} = -2\sin\theta\frac{d\theta}{dt} = -2(\sin\theta)(16\pi) = -32\pi\sin\theta$$
$$\frac{dy}{dt} = 2\cos\theta\frac{d\theta}{dt} = 2(\cos\theta)(16\pi) = 32\pi\cos\theta$$

At $\theta = \frac{\pi}{4}$:

$$\frac{dx}{dt} = -32\pi\ \sin\frac{\pi}{4} = -16\pi(\sqrt{2}) \approx -71.086 \text{ ft/sec}$$
$$\frac{dy}{dt} = 32\pi\ \cos\frac{\pi}{4} = 16\pi(\sqrt{2}) \approx 71.086 \text{ ft/sec}$$

At $\theta = \frac{\pi}{2}$:

$$\frac{dx}{dt} = -32\pi\sin\frac{\pi}{2} = -32\pi \approx -100.531 \text{ ft/sec}$$
$$\frac{dy}{dt} = 32\pi\cos\frac{\pi}{2} = 0 \text{ ft/sec}$$

At $\theta = \pi$:

$$\frac{dx}{dt} = -32\pi\ \sin\pi = 0 \text{ ft/sec}$$
$$\frac{dy}{dt} = 32\pi\ \cos\pi = -32\pi \approx -100.531 \text{ ft/sec}$$

46. (a) One possible answer:
$x = 30\cos\theta,\ y = 40 + 30\sin\theta$

(b) Since the ferris wheel makes one revolution every 10 sec, we may let $\theta = 0.2\pi t$ and we may write $x = 30\cos 0.2\pi t,\ y = 40 + 30\sin 0.2\pi t$. (This assumes that the ferris wheel revolves counterclockwise.)

In general:

$$\frac{dx}{dt} = -30(\sin 0.2\pi t)(0.2\pi) = -6\pi\sin 0.2\pi t$$
$$\frac{dy}{dt} = 30(\cos 0.2\pi t)(0.2\pi) = 6\pi\cos 0.2\pi t$$

At $t = 5$:

$$\frac{dx}{dt} = -6\pi\sin\pi = 0 \text{ ft/sec}$$
$$\frac{dy}{dt} = 6\pi\cos\pi = 6\pi(-1) \approx -18.850 \text{ ft/sec}$$

At $t = 8$:

$$\frac{dx}{dt} = -6\pi\sin 1.6\pi \approx 17.927 \text{ ft/sec}$$
$$\frac{dy}{dt} = 6\pi\cos 1.6\pi \approx 5.825 \text{ ft/sec}$$

47. (a)
$$\frac{dy}{dt} = \frac{d}{dt}(uv) = u\frac{dv}{dt} + v\frac{du}{dt}$$
$$= u(0.05v) + v(0.04u)$$
$$= 0.09uv$$
$$= 0.09y$$

Since $\frac{dy}{dt} = 0.09y$, the rate of growth of total production is 9% per year.

(b)
$$\frac{dy}{dt} = \frac{d}{dt}(uv) = u\frac{dv}{dt} + v\frac{du}{dt}$$
$$= u(0.03v) + v(-0.02u)$$
$$= 0.01uv$$
$$= 0.01y$$

The total production is increasing at the rate of 1% per year.

Quick Quiz Sections 4.4–4.6

1. B. $x_{n+1} = x_n - \frac{f(x)}{f'(x)}$

$$f(x) = x^3 + 2x - 1$$
$$f'(x) = 3x^2 + 2$$
$$x_2 = 1 - \frac{(1)^3 + 2(1) - 1}{3(1)^2 + 2} = \frac{3}{5}$$
$$x_3 = \frac{3}{5} - \frac{\left(\frac{3}{5}\right)^3 + 2\left(\frac{3}{5}\right) - 1}{3\left(\frac{3}{5}\right)^2 + 2} = 0.465$$

2. B. $z^2 = x^2 + y^2$

$$z = \sqrt{4^2 + 3^2} = 5$$
$$2z\frac{dz}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$$
$$5 = 4\left(3\frac{dy}{dt}\right) + 3\frac{dy}{dt}$$
$$\frac{dy}{dt} = \frac{1}{3}$$
$$\frac{dx}{dt} = 3\frac{dy}{dt} = 3\left(\frac{1}{3}\right) = 1$$

3. A. $x(t) = 70$

$$y(t) = 60t$$
$$z(t) = ((60t)^2 + 70^2)^{1/2}$$
$$\frac{dz}{dt} = \frac{1}{2}(3600t^2 + 4900)^{-1/2}(7200t)$$
$$\frac{dz}{dt} = \frac{7200(4)}{2(3600(4)^2 + 4900)^{1/2}}$$
$$\frac{dz}{dt} = 57.6$$

4. (a) $f(x) = \sqrt{x}$

$$x = 25$$
$$f'(25) = \frac{1}{2}(25)^{-1/2} = \frac{1}{10}$$
$$\sqrt{26} = 5 + \frac{1}{10}(26 - 25) = 5.1$$