5. f is differentiable for all x in the interior of its domain, or, in the interval $(-2, 2)$.

6. We require $x^2 - 1 \neq 0$, so the domain is $x \neq \pm 1$.

7. f is continuous for all x in the domain, or, for all $x \neq \pm 1$.

8. f is differentiable for all x in the domain, or, for all $x \neq \pm 1$.

9. $$7 = -2(-2) + C$$
$$7 = 4 + C$$
$$C = 3$$

10. $$-1 = (1)^2 + 2(1) + C$$
$$-1 = 3 + C$$
$$C = -4$$

Section 4.2 Exercises

1. (a) Yes.

(b) $f'(x) = \frac{d}{dx}(x^2 + 2x - 1) = 2x + 2$

$$2c + 2 = \frac{2-(-1)}{1-0} = 3$$
$$c = \frac{1}{2}.$$

2. (a) Yes.

(b) $f'(x) = \frac{d}{dx}x^{2/3} = \frac{2}{3}x^{-1/3}$

$$\frac{2}{3}c^{-1/3} = \frac{1-0}{1-0} = 1$$
$$c = \frac{8}{27}.$$

3. (a) No. There is a vertical tangent at $x = 0$.

4. (a) No. There is a corner at $x = 1$.

5. (a) Yes.

(b) $f'(x) = \frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$

$$\frac{1}{\sqrt{1-c^2}} = \frac{(\pi/2)-(-\pi/2)}{1-(-1)} = \frac{\pi}{2}$$
$$\sqrt{1-c^2} = \frac{2}{\pi}$$
$$c = \sqrt{1-4/\pi^2} \approx 0.771.$$

6. (a) Yes.

(b) $f'(x) = \frac{d}{dx}\ln(x-1) = \frac{1}{x-1}$

$$\frac{1}{c-1} = \frac{\ln 3 - \ln 1}{4-2}$$
$$c = \frac{4-2}{\ln 3 - \ln 1} + 1 \approx 2.820$$

7. (a) No. The function is discontinuous at $x = \frac{\pi}{2}$

8. (a) No. The split function is discontinuous at $x = 1$

9. (a) The secant line passes through $(0.5, f(0.5)) = (0.5, 2.5)$ and $(2, f(2)) = (2, 2.5)$, so its equation is $y = 2.5$.

(b) The slope of the secant line is 0, so we need to find c such that $f'(c) = 0$.

$$1 - c^{-2} = 0$$
$$c^{-2} = 1$$
$$c = 1$$
$$f(c) = f(1) = 2$$

The tangent line has slope 0 and passes through $(1, 2)$, so its equation is $y = 2$.

10. (a) The secant line passes through $(1, f(1)) = (1, 0)$ and $(3, f(3)) = (3, \sqrt{2})$, so its slope is

$$\frac{\sqrt{2}-0}{3-1} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}.$$

The equation is $y = \frac{1}{\sqrt{2}}(x-1) + 0$

or $y = \frac{1}{\sqrt{2}}x - \frac{1}{\sqrt{2}}$, or $y \approx 0.707x - 0.707$.

(b) We need to find c such that $f'(c) = \frac{1}{\sqrt{2}}$.

$$\frac{1}{2\sqrt{c-1}} = \frac{1}{\sqrt{2}}$$
$$2\sqrt{c-1} = \sqrt{2}$$
$$c - 1 = \frac{1}{2}$$
$$c = \frac{3}{2}$$
$$f(c) = f\left(\frac{3}{2}\right) = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$$

The tangent line has slope $\frac{1}{\sqrt{2}}$ and passes through $\left(\frac{3}{2}, \frac{1}{\sqrt{2}}\right)$. Its equation is $y = \frac{1}{\sqrt{2}}\left(x - \frac{3}{2}\right) + \frac{1}{\sqrt{2}}$ or

$y = \frac{1}{\sqrt{2}}x - \frac{1}{2\sqrt{2}}$, or $y \approx 0.707x - 0.354$.

11. Because the trucker's average speed was 79.5 mph, and by the Mean Value Theorem, the trucker must have been going that speed at least once during the trip.

12. Let $f(t)$ denote the temperature indicated after t seconds. We assume that $f'(t)$ is defined and continuous for $0 \le t \le 20$. The average rate of change is $10.6°$F/sec. Therefore, by the Mean Value Theorem, $f'(c) = 10.6°$ F/sec for some value of c in $[0, 20]$. Since the temperature was constant before $t = 0$, we also know that $f'(0) = 0°$ F/min. But f' is continuous, so by the Intermediate Value Theorem, the rate of change $f'(t)$ must have been $10.1°$F/sec at some moment during the interval.

13. Because its average speed was approximately 7.667 knots, and by the Mean Value Theorem, it must been going that speed at least once during the trip.

14. The runner's average speed for the marathon was approximately 11.909 mph. Therefore, by the Mean Value Theroem, the runner must have been going that speed at least once during the marathon. Since the initial speed and final speed are both 0 mph and the runner's speed is continuous, by the Intermediate Value Theorem, the runner's speed must have been 11 mph at least twice.

15. (a) $f'(x) = 5 - 2x$

Since $f'(x) > 0$ on $\left(-\infty, \frac{5}{2}\right)$, $f'(x) = 0$ at $x = \frac{5}{2}$, and $f'(x) < 0$ on $\left(\frac{5}{2}, \infty\right)$, we know that $f(x)$ has a local maximum at $x = \frac{5}{2}$. Since $f\left(\frac{5}{2}\right) = \frac{25}{4}$, the local maximum occurs at the point $\left(\frac{5}{2}, \frac{25}{4}\right)$. (This is also a global maximum.)

(b) Since $f'(x) > 0$ on $\left(-\infty, \frac{5}{2}\right)$, $f(x)$ is increasing on $\left(-\infty, \frac{5}{2}\right]$.

(c) Since $f'(x) < 0$ on $\left(\frac{5}{2}, \infty\right)$, $f(x)$ is decreasing on $\left[\frac{5}{2}, \infty\right)$.

16. (a) $g'(x) = 2x - 1$

Since $g'(x) < 0$ on $\left(-\infty, \frac{1}{2}\right)$, $g'(x) = 0$ at $x = \frac{1}{2}$, and $g'(x) > 0$ on $\left(\frac{1}{2}, \infty\right)$, we know that $g(x)$ has a local minimum at $x = \frac{1}{2}$.

Since $g\left(\frac{1}{2}\right) = -\frac{49}{4}$, the local minimum occurs at the point $\left(\frac{1}{2}, -\frac{49}{4}\right)$. (This is also a global minimum.)

(b) Since $g'(x) > 0$ on $\left(\frac{1}{2}, \infty\right)$, $g(x)$ is increasing on $\left[\frac{1}{2}, \infty\right)$.

(c) Since $g'(x) < 0$ on $\left(-\infty, \frac{1}{2}\right)$, $g(x)$ is decreasing on $\left(-\infty, \frac{1}{2}\right]$.

17. (a) $h'(x) = -\frac{2}{x^2}$

Since $h'(x)$ is never zero and is undefined only where $h(x)$ is undefined, there are no critical points. Also, the domain $(-\infty, 0) \cup (0, \infty)$ has no endpoints. Therefore, $h(x)$ has no local extrema.

(b) Since $h'(x)$ is never positive, $h(x)$ is not increasing on any interval.

(c) Since $h'(x) < 0$ on $(-\infty, 0) \cup (0, \infty)$, $h(x)$ is decreasing on $(-\infty, 0)$ and on $(0, \infty)$.

18. (a) $k'(x) = -\frac{2}{x^3}$

Since $k'(x)$ is never zero and is undefined only where $k(x)$ is undefined, there are no critical points. Also, the domain $(-\infty, 0) \cup (0, \infty)$ has no endpoints. Therefore, $k(x)$ has no local extrema.

(b) Since $k'(x) > 0$ on $(-\infty, 0)$, $k(x)$ is increasing on $(-\infty, 0)$.

(c) Since $k'(x) < 0$ on $(0, \infty)$, $k(x)$ is decreasing on $(0, \infty)$.

19. (a) $f'(x) = 2e^{2x}$

Since $f'(x)$ is never zero or undefined, and the domain of $f(x)$ has no endpoints, $f(x)$ has no extrema.

(b) Since $f'(x)$ is always positive, $f(x)$ is increasing on $(-\infty, \infty)$.

(c) Since $f'(x)$ is never negative, $f(x)$ is not decreasing on any interval.

20. (a) $f'(x) = -0.5e^{-0.5x}$

Since $f'(x)$ is never zero or undefined, and the domain of $f(x)$ has no endpoints, $f(x)$ has no extrema.

(b) Since $f'(x)$ is never positive, $f(x)$ is not increasing on any interval.

(c) Since $f'(x)$ is always negative, $f(x)$ is decreasing on $(-\infty, \infty)$.

21. (a) $y' = -\frac{1}{2\sqrt{x+2}}$

In the domain $[-2, \infty)$, y' is never zero and is undefined only at the endpoint $x = -2$. The function y has a local maximum at $(-2, 4)$. (This is also a global maximum.)

(b) Since y' is never positive, y is not increasing on any interval.

(c) Since y' is negative on $(-2, \infty)$, y is decreasing on $[-2, \infty)$.

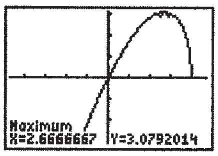
22. **(a)** $y' = 4x^3 - 20x = 4x(x+\sqrt{5})(x-\sqrt{5})$

The function has critical points at $x = -\sqrt{5}$, $x = 0$, and $x = \sqrt{5}$. Since $y' < 0$ on $(-\infty, -\sqrt{5})$ and $(0, \sqrt{5})$ and $y' > 0$ on $(-\sqrt{5}, 0)$ and $(\sqrt{5}, \infty)$, the points at $x = \pm\sqrt{5}$ are local minima and the point at $x = 0$ is a local maximum. Thus, the function has a local maximum at $(0, 9)$ and local minima at $(-\sqrt{5}, -16)$ and $(\sqrt{5}, -16)$. (These are also global minima.)

(b) Since $y' > 0$ on $(-\sqrt{5}, 0)$ and $(\sqrt{5}, \infty)$, y is increasing on $[-\sqrt{5}, 0]$ and $[\sqrt{5}, \infty)$.

(c) Since $y' > 0$ on $(-\infty, -\sqrt{5})$ and $(0, \sqrt{5})$, y is decreasing on $(-\infty, -\sqrt{5}]$ and $[0, \sqrt{5}]$.

23.



$[-4.7, 4.7]$ by $[-3.1, 3.1]$

(a) $f'(x) = x \cdot \dfrac{1}{2\sqrt{4-x}}(-1) + \sqrt{4-x}$

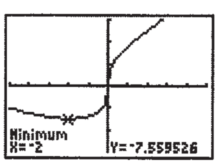
$= \dfrac{-3x+8}{2\sqrt{4-x}}$

The local extrema occur at the critical point $x = \dfrac{8}{3}$ and at the endpoint $x = 4$. There is a local (and absolute) maximum at $\left(\dfrac{8}{3}, \dfrac{16}{3\sqrt{3}}\right)$ or approximately $(2.67, 3.08)$, and a local minimum at $(4, 0)$.

(b) Since $f'(x) > 0$ on $\left(-\infty, \dfrac{8}{3}\right)$, $f(x)$ is increasing on $\left(-\infty, \dfrac{8}{3}\right]$.

(c) Since $f'(x) < 0$ on $\left(\dfrac{8}{3}, 4\right)$, $f(x)$ is decreasing on $\left[\dfrac{8}{3}, 4\right]$.

24.



$[-5, 5]$ by $[-15, 15]$

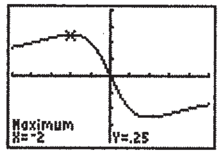
(a) $g'(x) = x^{1/3}(1) + \dfrac{1}{3}x^{-2/3}(x+8) = \dfrac{4x+8}{3x^{2/3}}$

The local extrema can occur at the critical points $x = -2$ and $x = 0$, but the graph shows that no extrema occurs at $x = 0$. There is a local (and absolute) minimum at $(-2, -6\sqrt[3]{2})$ or approximately $(-2, -7.56)$.

(b) Since $g'(x) > 0$ on the intervals $(-2, 0)$ and $(0, \infty)$, and $g(x)$ is continuous at $x = 0$, $g(x)$ is increasing on $[-2, \infty)$.

(c) Since $g'(x) < 0$ on the interval $(-\infty, -2)$, $g(x)$ is decreasing on $(-\infty, -2]$.

25.



$[-5, 5]$ by $[-0.4, 0.4]$

(a) $h'(x) = \dfrac{(x^2+4)(-1) - (-x)(2x)}{(x^2+4)^2} = \dfrac{x^2-4}{(x^2+4)^2}$

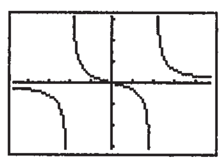
$= \dfrac{(x+2)(x-2)}{(x^2+4)^2}$

The local extrema occur at the critical points, $x = \pm 2$. There is a local (and absolute) maximum at $\left(-2, \dfrac{1}{4}\right)$ and a local (and absolute) minimum at $\left(2, -\dfrac{1}{4}\right)$.

(b) Since $h'(x) > 0$ on $(-\infty, -2)$ and $(2, \infty)$, $h(x)$ is increasing on $(-\infty, -2]$ and $[2, \infty)$.

(c) Since $h'(x) < 0$ on $(-2, 2)$, $h(x)$ is decreasing on $[-2, 2]$.

26.



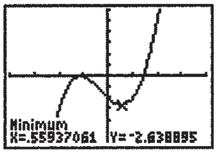
$[-4.7, 4.7]$ by $[-3.1, 3.1]$

(a) $k'(x) = \dfrac{(x^2-4)(1) - x(2x)}{(x^2-4)^2} = -\dfrac{x^2+4}{(x^2-4)^2}$

Since $k'(x)$ is never zero and is undefined only where $k(x)$ is undefined, there are no critical points. Since there are no critical points and the domain includes no endpoints, $k(x)$ has no local extrema.

(b) Since $k'(x)$ is never positive, $k(x)$ is not increasing on any interval.

(c) Since $k'(x)$ is negative wherever it is defined, $k(x)$ is decreasing on each interval of its domain; on $(-\infty, -2)$, $(-2, 2)$, and $(2, \infty)$.

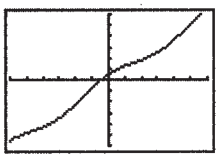
27. 

[−4, 4] by [−6, 6]

(a) $f'(x) = 3x^2 - 2 + 2\sin x$
Note that $3x^2 - 2 > 2$ for $|x| \geq 1.2$ and $|2 \sin x| \leq 2$ for all x, so $f'(x) > 0$ for $|x| \geq 1.2$. Therefore, all critical points occur in the interval (–1.2, 1.2), as suggested by the graph. Using grapher techniques, there is a local maximum at approximately (–1.126, –0.036), and a local minimum at approximately (0.559, –2.639).

(b) $f(x)$ is increasing on the intervals $(-\infty, -1.126]$ and $[0.559, \infty)$, where the interval endpoints are approximate.

(c) $f(x)$ is decreasing on the interval [–1.126, 0.559], where the interval endpoints are approximate.

28. 

[−6, 6] by [−12, 12]

(a) $g'(x) = 2 - \sin x$
Since $1 \leq g'(x) \leq 3$ for all x, there are no critical points. Since there are no critical points and the domain has no endpoints, there are no local extrema.

(b) Since $g'(x) > 0$ for all x, $g(x)$ is increasing on $(-\infty, \infty)$.

(c) Since $g'(x)$ is never negative, $g(x)$ is not decreasing on any interval.

29. $f(x) = \dfrac{x^2}{2} + C$

30. $f(x) = 2x + C$

31. $f(x) = x^3 - x^2 + x + C$

32. $f(x) = -\cos x + C$

33. $f(x) = e^x + C$

34. $f(x) = \ln(x-1) + C$

35. $f(x) = \dfrac{1}{x} + C,\ x > 0$

$$f(2) = 1$$
$$\frac{1}{2} + C = 1$$
$$C = \frac{1}{2}$$
$$f(x) = \frac{1}{x} + \frac{1}{2},\ x > 0$$

36. $f(x) = x^{1/4} + C$

$$f(1) = -2$$
$$1^{1/4} + C = -2$$
$$1 + C = -2$$
$$C = -3$$
$$f(x) = x^{1/4} - 3$$

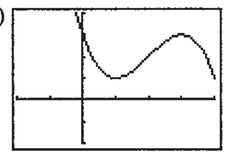
37. $f(x) = \ln(x+2) + C$

$$f(-1) = 3$$
$$\ln(-1+2) + C = 3$$
$$0 + C = 3$$
$$C = 3$$
$$f(x) = \ln(x+2) + 3$$

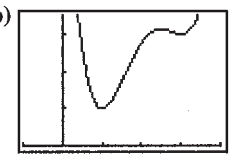
38. $f(x) = x^2 + x - \sin x + C$

$$f(0) = 3$$
$$0 + C = 3$$
$$C = 3$$
$$f(x) = x^2 + x - \sin x + 3$$

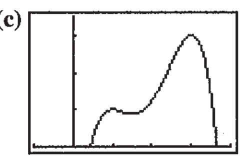
39. Possible answers:

(a) 

[–2, 4] by [–2, 4]

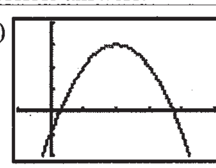
(b) 

[–1, 4] by [0, 3.5]

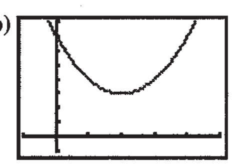
(c) 

[–1, 4] by [0, 3.5]

40. Possible answers:

(a) 

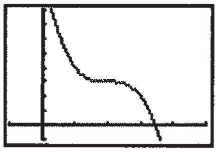
[−1, 5] by [−2, 4]

(b) 

[−1, 5] by [−1, 8]

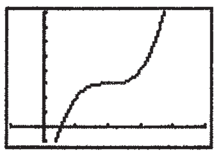
40. Continued

(c)



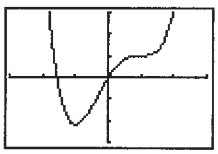
[−1, 5] by [−1, 8]

(d)



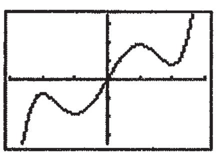
[−1, 5] by [−1, 8]

41. One possible answer:



[−3, 3] by [−15, 15]

42. One possible answer:



[−3, 3] by [−70, 70]

43. (a) Since $v'(t) = 1.6$, $v(t) = 1.6t + C$. But $v(0) = 0$, so $C = 0$ and $v(t) = 1.6t$. Therefore, $v(30) = 1.6(30) = 48$. The rock will be going 48 m/sec.

(b) Let $s(t)$ represent position.
Since $s'(t) = v(t) = 1.6t$, $s(t) = 0.8t^2 + D$. But $s(0) = 0$, so $D = 0$ and $s(t) = 0.8t^2$. Therefore, $s(30) = 0.8(30)^2 = 720$. The rock travels 720 meters in the 30 seconds it takes to hit bottom, so the bottom of the crevasse is 720 meters below the point of release.

(c) The velocity is now given by $v(t) = 1.6t + C$, where $v(0) = 4$. (Note that the sign of the initial velocity is the same as the sign used for the acceleration, since both act in a downward direction.) Therefore, $v(t) = 1.6t + 4$, and $s(t) = 0.8t^2 + 4t + D$, where $s(0) = 0$ and so $D = 0$. Using $s(t) = 0.8t^2 + 4t$ and the known crevasse depth of 720 meters, we solve $s(t) = 720$ to obtain the positive solution $t \approx 27.604$, and so $v(t) = v(27.604) = 1.6(27.604) + 4 \approx 48.166$. The rock will hit bottom after about 27.604 seconds, and it will be going about 48.166 m/sec.

44. (a) We assume the diving board is located at $s = 0$ and the water at $s = 10$, so that downward velocities are positive. The acceleration due to gravity is 9.8 m/sec^2, so $v'(t) = 9.8$ and $v(t) = 9.8t + C$. Since $v(0) = 0$, we have $v(t) = 9.8t$. Then the position is given by $s(t)$ where $s'(t) = v(t) = 9.8t$, so $s(t) = 4.9t^2 + D$. Since $s(0) = 0$, we have $s(t) = 4.9t^2$. Solving $s(t) = 10$ gives $t^2 = \frac{10}{4.9} = \frac{100}{49}$, so the positive solution is $t = \frac{10}{7}$. The velocity at this time is $v\left(\frac{10}{7}\right) = 9.8\left(\frac{10}{7}\right) = 14$ m/sec.

(b) Again $v(t) = 9.8t + C$, but this time $v(0) = -2$ and so $v(t) = 9.8t - 2$. Then $s'(t) = 9.8t - 2$, so $s(t) = 4.9t^2 - 2t + D$. Since $s(0) = 0$, we have $s(t) = 4.9t^2 - 2t$. Solving $s(t) = 10$ gives the positive solution $t = \frac{2 + 10\sqrt{2}}{9.8} \approx 1.647$ sec.
The velocity at this time is
$v\left(\frac{2+10\sqrt{2}}{9.8}\right) = 9.8\left(\frac{2+10\sqrt{2}}{9.8}\right) - 2 = 10\sqrt{2}$ m/sec or about 14.142 m/sec.

45. Because the function is not continuous on [0, 1]. The function does not satisfy the hypotheses of the Mean Value Theorem, and so it need not satisfy the conclusion of the Mean Value Theorem.

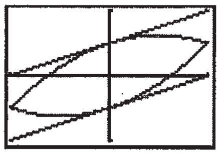
46. Because the Mean Value Theorem applies to the function $y = \sin x$ on any interval, and $y = \cos x$ is the derivative of $\sin x$. So, between any two zeros of $\sin x$, its derivative, $\cos x$, must be zero at least once.

47. $f(x)$ must be zero at least once between a and b by the Intermediate Value Theorem. Now suppose that $f(x)$ is zero twice between a and b. Then by the Mean Value Theorem, $f'(x)$ would have to be zero at least once between the two zeros of $f(x)$, but this can't be true since we are given that $f'(x) \neq 0$ on this interval. Therefore, $f(x)$ is zero once and only once between a and b.

48. Let $f(x) = x^4 + 3x + 1$. Then $f(x)$ is continuous and differentiable everywhere. $f'(x) = 4x^3 + 3$, which is never zero between $x = -2$ and $x = -1$. Since $f(-2) = 11$ and $f(-1) = -1$, exercise 47 applies, and $f(x)$ has exactly one zero between $x = -2$ and $x = -1$.

49. Let $f(x) = x + \ln(x + 1)$. Then $f(x)$ is continuous and differentiable everywhere on [0, 3]. $f'(x) = 1 + \frac{1}{x+1}$, which is never zero on [0, 3]. Now $f(0) = 0$, so $x = 0$ is one solution of the equation. If there were a second solution, $f(x)$ would be zero twice in [0, 3], and by the Mean Value Theorem, $f'(x)$ would have to be zero somewhere between the two zeros of $f(x)$ .But this can't happen, since $f'(x)$ is never zero on [0, 3]. Therefore, $f(x) = 0$ has exactly one solution in the interval [0, 3].

50. Consider the function $k(x) = f(x) - g(x)$. $k(x)$ is continuous and differentiable on $[a, b]$, and since $k(a) = f(a) - g(a) = 0$ and $k(b) = f(b) - g(b) = 0$, by the Mean Value Theorem, there must be a point c in (a, b) where $k'(c) = 0$. But since $k'(c) = f'(c) - g'(c)$, this means that $f'(c) = g'(c)$, and c is a point where the graphs of f and g have parallel or identical tangent lines.



$(-1, 1)$ by $[-2, 2]$

51. False. For example, the function x^3 is increasing on $(-1, 1)$, but $f'(0) = 0$.

52. True. In fact, f is increasing on $[a, b]$ by Corollary 1 to the Mean Value Theorem.

53. A. $f'(x) = \dfrac{\frac{1}{2} - 1}{\frac{\pi}{3}} = -\dfrac{3}{2\pi}$.

54. B. $f'(x) = e^{x^3-6x^2+8}(3x^2 - 12x) = e^{x^3-6x^2+8}(3x)(x-4)$, which is negative only when x is between 0 and 4.

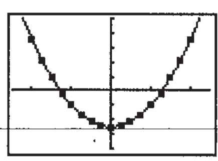
55. E. $\dfrac{d}{dx}(2\sqrt{x} - 10)$

$= \dfrac{2}{2\sqrt{x}} = \dfrac{1}{\sqrt{x}}$.

56. D. $x^{3/5}$ is not differentiable at $x = 0$.

57. (a) Increasing: $[-2, -1.3]$ and $[1.3, 2]$;
decreasing: $[-1.3, 1.3]$;
local max: $x \approx -1.3$
local min: $x \approx 1.3$

(b) Regression equation: $y = 3x^2 - 5$



$[-2.5, 2.5]$ by $[-8, 10]$

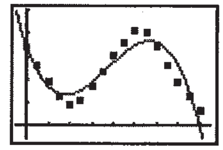
(c) Since $f'(x) = 3x^2 - 5$, we have $f(x) = x^3 - 5x + C$.

But $f(0) = 0$, so $C = 0$. Then $f(x) = x^3 - 5x$.

58. (a) Toward: $0 < t < 2$ and $5 < t < 8$; away: $2 < t < 5$

(b) A local extremum in this problem is a time/place where Priya changes the direction of her motion.

(c) Regression equation:
$y = -0.0820x^3 + 0.9162x^2 - 2.5126x + 3.3779$



$[-0.5, 8.5]$ by $[-0.5, 5]$

(d) Using the unrounded values from the regression equation, we obtain $f'(t) = -0.2459t^2 + 1.8324t - 2.5126$. According to the regression equation, Priya is moving toward the motion detector when $f'(t) < 0$ $(0 < t < 1.81$ and $5.64 < t < 8)$, and away from the detector when $f'(t) > 0$ $(1.81 < t < 5.64)$.

59. $\dfrac{f(b) - f(a)}{b - a} = \dfrac{\frac{1}{b} - \frac{1}{a}}{b - a} = -\dfrac{1}{ab}$

$f'(c) = -\dfrac{1}{c^2}$, so $-\dfrac{1}{c^2} = -\dfrac{1}{ab}$ and $c^2 = ab$.

Thus, $c = \sqrt{ab}$.

60. $\dfrac{f(b) - f(a)}{b - a} = \dfrac{b^2 - a^2}{b - a} = b + a$

$f'(c) = 2c$, so $2c = b + a$ and $c = \dfrac{a + b}{2}$.

61. By the Mean Value Theorem, $\sin b - \sin a = (\cos c)(b - a)$ for some c between a and b. Taking the absolute value of both sides and using $|\cos c| \le 1$ gives the result.

62. Since differentiability implies continuity, we can apply the Mean Value Theorem to f on $[a, b]$.

Since $f(b) < f(a)$, $\dfrac{f(b) - f(a)}{b - a}$ is negative, and hence $f'(x)$ must be negative at some point between a and b.

63. Let $f(x)$ be a monotonic function defined on an interval D. For any two values in D, we may let x_1 be the smaller value and let x_2 be the larger value, so $x_1 < x_2$. Then either $f(x_1) < f(x_2)$ (if f is increasing), or $f(x_1) > f(x_2)$ (if f is decreasing), which means $f(x_1) \ne f(x_2)$. Therefore, f is one-to-one.

Section 4.3 Connecting f' and f'' with the Graph of f (pp. 205–218)

Exploration 1 Finding f from f'

1. Any function $f(x) = x^4 - 4x^3 + C$ where C is a real number. For example, let $C = 0, 1, 2$. Their graphs are all vertical shifts of each other.

2. Their behavior is the same as the behavior of the function f of Example 8.